Surprising absence of strong homonuclear coupling at low magnetic field explored by two-field NMR spectroscopy

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Abstract

Strong coupling of nuclear spins, which is achieved when their scalar coupling $2\pi J$ is greater than or comparable to the difference $\delta \omega$ in their Larmor precession frequencies in an external magnetic field, gives rise to efficient coherent longitudinal polarization transfer. The strong-coupling regime can be achieved when the external magnetic field is sufficiently low, as $\delta \omega$ is reduced proportional to the field strength. In the present work, however, we demonstrate that in heteronuclear spin systems these simple arguments may not hold, since heteronuclear spin-spin interactions alter the $\delta \omega$ value. The experimental method that we use is two-field NMR (Nuclear Magnetic Resonance), exploiting sample shuttling between a high field, at which NMR spectra are acquired, and low field, where strong couplings are expected, at which NMR pulses can be applied to affect the spin dynamics. By using this technique, we generate zero-quantum spin coherences by means of non-adiabatic passage through a level anti-crossing and study their evolution at low field. Such zero-quantum coherences mediate the polarization transfer under strong coupling conditions. Experiments performed with an $^{13}$C labelled amino acid clearly show that the coherent polarization transfer at low field is pronounced in the $^{13}$C-spin subsystem under proton decoupling. However, in the absence of proton decoupling, polarization transfer by coherent processes is dramatically reduced, demonstrating that heteronuclear spin-spin interactions suppress the strong coupling regime even when the external field is low. A theoretical model is presented, which can model the reported experimental results.

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I. Introduction

The topological and conformational information provided by scalar couplings lies at the foundation of the analytical power of NMR spectroscopy (Ernst et al., 1987; Keeler, 2005; Levitt, 2008; Cavanagh, 2007). The strong coupling case is encountered when scalar coupling constants are not negligible with respect to the difference of resonance frequency between the coupled spins (Keeler, 2005). Understanding strong scalar couplings and their spectral signature was essential when NMR was introduced for chemical analysis, which was typically performed at magnetic fields considered today as low (Bodenhausen et al., 1977; Pfändler and Bodenhausen, 1987). Modern high-field NMR is widely based on the exploitation of weak scalar couplings, so that strong scalar couplings have remained a nuisance, in particular in aromatic spin systems (Vallurupalli et al., 2007; Foroozandeh et al., 2014). Recently, the development and availability of benchtop NMR spectrometers operating at low or moderate magnetic fields (Grootveld et al., 2019), has revived the interest in the understanding of strong scalar couplings in conventional NMR.

Contrarily to conventional NMR, NMR at near-zero or ultralow magnetic fields (ZULF-NMR), explores the benefits of NMR in the strong scalar-coupling regime. At such magnetic fields, typically smaller than 1 \( \mu \)T, scalar coupling interactions dominate all Zeeman interaction and dictate the eigenstates of spin systems and transition energies obtained in spectra (Ledbetter et al., 2011; Tayler et al., 2017; Blanchard and Budker, 2016). However, for homonuclear couplings, the transition between the weak- and strong-coupling regimes occurs in a range of magnetic fields, where the Zeeman interaction is still dominant (Ivanov et al., 2006; Ivanov et al., 2008; Ivanov et al., 2014; Appelt et al., 2010). This transition between weak and strong couplings can be investigated by varying the magnetic field applied to the sample on a high-field magnet, which is usually performed by moving the sample through the stray field with a shuttle system (Roberts and Redfield, 2004b, a; Redfield, 2012; Wagner et al., 1999; Bryant and Korb, 2005; Goddard et al., 2007; Chou et al., 2016; Chou et al., 2017; Charlier et al., 2013; Cousin et al., 2016a; Cousin et al., 2016b; Zhukov et al., 2018; Kiryutin et al., 2016). These studies have highlighted the effects of level anti-crossings (LACs) (Miesel et al., 2006; Ivanov et al., 2014). When the passage through a LAC is slow, the transition is adiabatic and the population of eigenstates is smoothly converted to the new eigenstates.

When the transition is fast, coherences can be generated between the new eigenstates and time-oscillations of the population of high-field eigenstates can be observed (Pravdivtsev et al., 2013; Kiryutin et al., 2013). This phenomenon has been observed on a variety of homonuclear spin systems. Heteronuclear scalar couplings have been shown to alter LACs in homonuclear spin systems (Korchak et al., 2012); yet, the properties of such heteronuclear couplings on LACs are not fully understood, in particular, in spin systems with extensive networks of homo- and heteronuclear scalar couplings.

Here, we investigate the effect of heteronuclear scalar couplings on LACs in a spin system typical of biomolecular NMR, a uniformly carbon-13 labeled amino acid (leucine), which combines extensive networks of homo- and heteronuclear scalar couplings. Essentially, we exploit the ability to apply composite pulse decoupling on our two-field NMR spectrometer (Cousin et al., 2016a) to switch on and off heteronuclear scalar couplings at low magnetic field. We demonstrate that heteronuclear scalar couplings alter LACs by sustaining the weak-coupling regime in a carbon-13 homonuclear spin system. Composite pulse decoupling at low magnetic field restores the strong scalar coupling regime in the carbon-13 nuclei of the isopropyl group of leucine at 0.33 T. Our results identify how heteronuclear couplings alter homonuclear couplings at low magnetic fields, which could be exploited in low-field NMR methodology and may be considered in further developments of total correlation spectroscopy (TOCSY) (Braunschweiler and Ernst, 1983) mixing sequences in high-field NMR.

II. Methods

A. Sample preparation
Experiments have been performed using the following sample: 76 mM 99% enriched $^{13}$C,$^{15}$N labeled L-81 leucine (Leu) in 90% $^2$H$_2$O 10% D$_2$O solution. $^{13}$C,$^{15}$N enriched L-leucine were purchased from Sigma-Aldrich and used as it stands. $^{13}$C-NMR spectrum of the labelled Leu molecule is shown in Figure 1. We also show separately the signals of the individual carbon nuclei. Broadband proton decoupling was used to simplify the spectrum. Here, we will focus on a three-spin system, formed by the C$^\gamma$ and two C$^\delta_n$ nuclei of the isopropyl moiety. We will study polarization transfer in this subsystem upon fast switch of the external magnetic field obtained by a transfer of the sample though the stray field of a high-field NMR magnet.

![Figure 1. Structure of $^{13}$C,$^{15}$N L-leucine and 100.62 MHz $^{13}$C-NMR spectrum under broadband $^1$H decoupling. Signal of each carbon nuclei is also shown separately. The multiplet structure in the spectrum is due to non-decoupled $^{13}$C-$^{13}$C and $^{13}$C-$^{15}$N scalar interactions.](https://doi.org/10.5194/mr-2020-14)

**B. Field-cycling NMR experiments**

NMR experiments were performed on a two-field NMR spectrometer (Cousin et al., 2016a) with fast sample shuttling (Charlier et al., 2013). The high field $B_{HF} = 14.1$ T is the detection field of a 600 MHz NMR spectrometer while the low field is $B_{LF} = 0.33$ T. The magnetic field in the low-field centre is sufficiently homogeneous (inhomogeneities of the order of 10 ppm) so that radiofrequency (RF) pulses can be applied by using a triple-resonance NMR probe, as described previously (Cousin et al., 2016a).

Field-cycling NMR experiments were run according to the pulse sequences depicted in Figure 2. First, a non-equilibrium state is generated at $B_{HF}$ by applying a selective NMR pulse (shaped ReBurp pulse (Geen and Freeman, 1991)) to the C$^{62}$ nucleus (the pulse duration was 46.4 ms, the peak RF-field amplitude was adjusted to cover ca. 100 Hz bandwidth around the center of C$^{62}$ signal). To improve the selectivity of the pulse, simultaneous proton decoupling was used, which reduces multiplet overlay in the carbon-13 NMR spectrum. Following this preparation, the sample was shuttled from the high-field center to the low-field center $B_{HF} \rightarrow B_{LF}$ with a duration $t_1 = 110$ ms. The field jump is fast enough to be non-adiabatic and it is aimed to excite a spin coherence. Subsequently, the coherence evolves at $B_{LF}$ during a variable time period $\tau$. The shuttle transfer back to the high-field center leads to a second field jump $B_{LF} \rightarrow B_{HF}$ with a duration $t_2 = 95$ ms. This second non-adiabatic field jump to $B_{HF}$ converts the coherence into a population difference. Detection is performed after a $\pi/2$ pulse on the carbon-13 channel in the presence of proton decoupling. We perform two types of experiments, in which the carbon spin coherence (zero-quantum coherence, ZQC) evolves at $B_{LF}$ in the absence (see Figure 2A) and in the presence (see Figure 2B) of proton composite-pulse decoupling. Decoupling at $B_{LF}$ has been performed using composite pulse decoupling with the WALTZ-64 supercycle (Shaka et al., 1983) at low field on the proton RF-channel (operating at 14 MHz corresponding to the proton NMR frequency at 0.33 T). The $\tau$-dependence of
polarization is expected to be oscillatory, due to the coherent polarization exchange within the expectedly strongly coupled system of the C' and two Cδ carbon-13 nuclei.

**Figure 2.** Experimental protocols of field-cycling NMR experiments without 1H decoupling at the low field (A) and with 24 kHz WALTZ-64 1H decoupling at the low field (B). Details of the experiments: 0.56 W WALTZ-64 composite pulse decoupling on the proton channel was applied at \( B_{1} \) during 100 ms prior to a selective 180-degree pulse, in order to enhance \(^{13}\)C polarization by the nuclear Overhauser effect. The sample shuttle times, \( t_{1} \) and \( t_{2} \), were 80 ms and 120 ms, respectively. Selective inversion was performed with a ReBurp pulse (Geen and Freeman, 1991) with a duration of 46.4 ms at the \(^{13}\)C resonant frequency covering ca. 100 Hz bandwidth. The delay \( \tau \) at low field was varied with a 5 ms step. After sample transfer to high field, a hard 90-degree pulse generated \(^{13}\)C transverse magnetization; FID acquisition was done during 1.56 s under 2.7 kHz WALTZ-64 proton decoupling.

### III. Theory

#### A. Polarization transfer in a 3-spin system

In this subsection, we provide a theoretical description of the field-cycling NMR experiments. First, we present the analytical treatment of polarization transfer among two nuclei of the same kind, here spin \( I_{1} \) and spin \( I_{2} \) (e.g. two carbon-13 nuclei), in the presence of a third spin \( S \), which can be a heteronucleus (e.g. here a proton). This is the minimal system allowing us to detail the effect of a heteronucleus on polarization transfer among strongly coupled spins. We assume that spins \( I_{1} \) and \( I_{2} \) are in strong coupling conditions, meaning that the difference, \( \delta \omega \), in their Zeeman interaction frequencys with the external field is smaller than or comparable to the scalar-coupling constant, \( 2\pi J_{12} \), between them. When the strong coupling regime is achieved, the zero-quantum part of the scalar coupling, given by the operator \( \{I_{1} I_{2} - I_{1} I_{2}\} \), becomes active, giving rise to flips and flops of spins \( I_{1} \) and \( I_{2} \). The couplings to the third spin \( S \), \( J_{13} \) and \( J_{23} \), are assumed to be unequal (otherwise coupling to the proton give rise to an identical shift of the NMR frequencies of spins 1 and 2 and does not modify the eigenstates of this subsystem). The Hamiltonian of the spin system can be written as follows (in \( \hbar \) units):
\[ \mathcal{H}_{CCH} = \omega_1 I_{1z} - \omega_2 I_{2z} - \omega_3 \hat{S}_z + 2\pi f_{12} (\mathbf{I}_1 \cdot \mathbf{I}_2) + 2\pi f_{13} I_{1z}\hat{S}_z + 2\pi f_{23} I_{2z}\hat{S}_z \]  

(1)

Here \( I_1, I_2 \) and \( \hat{S} \) are the spin operators; \( \omega_1, \omega_2 \) and \( \omega_3 \) stand for the NMR frequencies of the corresponding nuclei. We assume that the heteronucleus \( S \) is coupled weakly to \( I \) spins due to the large difference in their NMR frequencies, i.e., \( |\omega_1 - \omega_3|, |\omega_2 - \omega_3| \gg |\omega_1 - \omega_2|, 2\pi f_{13}, 2\pi f_{23} \), and keep only the secular part of the heteronuclear coupling Hamiltonian.

In the present case, the nuclear magnetic number, \( m_S \), of spin \( S \) is a “good quantum number”, which is conserved because \( \hat{S}_z \) commutes with the Hamiltonian. For this reason, it is possible to find the solution for the spin dynamics of spins \( I_1 \) and \( I_2 \) for two separate cases, which corresponds to the two different values of \( m_S \), being \( \pm \frac{1}{2} \) and \( \pm \frac{1}{2} \), i.e., spin \( S \) is in the “spin-up” \( |a\rangle \) state or “spin-down” \( |b\rangle \) state. In each case, the Hamiltonian of the carbon subsystem is as follows:

\[ \mathcal{H}_{CC} = -(\omega_1 - 2\pi f_{13} S_z) I_{1z} - (\omega_2 - 2\pi f_{23} S_z) I_{2z} + 2\pi f_{12} (\mathbf{I}_1 \cdot \mathbf{I}_2) \]  

(2)

Hence, in the Hamiltonian given by eq. (1) we replace the \( \hat{S}_z \) operator by the \( m_S \) value, which is \( \pm \frac{1}{2} \). Hence, the \( \delta \omega \) value is modified and it depends on the \( m_S \) value:

\[ \delta \omega_{\pm} = (\omega_1 - \omega_2) \mp \pi (f_{13} - f_{23}) = \delta \omega \mp \pi \cdot \delta J \]  

(3)

The eigenstates of the subsystem of spin 1 and spin 2 are

\[
\begin{align*}
|1\rangle &= |\alpha\alpha\rangle, \\
|2\rangle_{\pm} &= \cos \theta_{\pm} |\alpha\beta\rangle + \sin \theta_{\pm} |\beta\alpha\rangle \\
|3\rangle_{\pm} &= -\sin \theta_{\pm} |\alpha\beta\rangle + \cos \theta_{\pm} |\beta\alpha\rangle, \\
|4\rangle &= |\beta\beta\rangle
\end{align*}
\]

(4)

Here the “mixing angle” is given by the values of \( \delta \omega_{\pm} \) and \( f_{12} \): \( \tan 2\theta_{\pm} = 2\pi f_{12}/\delta \omega_{\pm} \). When \( \delta \omega_{\pm} \) approaches zero, the mixing angle goes to \( \frac{\pi}{4} \) meaning that the eigenstates become singlet and triplet states: the spins are strongly coupled. When \( \delta \omega_{\pm} \) is much greater than the coupling, the eigenstates are obviously the Zeeman states.

Figure 3. Polarization transfer among two strongly coupled nuclei (a) in the absence and (b) in the presence (bottom) of a heteronucleus. Here, we present the time dependence of \( \langle I_{1z} \rangle \) (black solid lines) and \( \langle I_{2z} \rangle \) (red dashed lines), normalized to the initial value of \( \langle I_{1z} \rangle \). The density operator at time \( t = 0 \) is \( \sigma_0 = I_{1z} \). Parameters of the simulation were \( \delta \omega/2\pi = 10 \text{ Hz}, f_{12} = 30 \text{ Hz}, \) and (a) \( \delta J = 0 \text{ Hz} \) and (b) \( \delta J = 100 \text{ Hz} \).
Even in this simple system, it is clear that the condition $|\omega_1 - \omega_2| \ll 2\pi f_{12}$ is not sufficient to guarantee strong coupling of the two carbons. Indeed, when $\delta J$ is greater than $\delta \omega$ and $2\pi f_{12}$ the carbon spins become weakly coupled in the two sub-ensembles, corresponding to $m_S = \pm \frac{1}{2}$.

How do heteronuclear couplings affect polarization transfer in the carbon system? We assume that at $t = 0$ one of the spins has polarization $I_{1z} = P_0$ and the other spin is not polarized, $I_{2z} = 0$. Hereafter, it is convenient to use normalization $P_0 = 1$. The state of the spin system is then given by the density operator

$$\sigma_0 = I_{1z}$$  \hspace{1cm} (5)

As shown previously (Ivanov et al., 2006), in the two-spin system of $I_1$ and $I_2$, in the absence of coupling to any other spin, the polarization evolves with time as follows:

$$\langle I_{1z}(t) \rangle = 1 - \sin^2 \theta \left( \frac{1 - \cos(\omega_{Q2} t)}{2} \right), \quad \langle I_{2z}(t) \rangle = \sin^2 \theta \left( \frac{1 - \cos(\omega_{Q2} t)}{2} \right)$$  \hspace{1cm} (6)

where $\tan 2\theta = 2\pi f_{12}/\delta \omega$ and the oscillation frequency $\omega_{Q2} = \sqrt{\delta \omega^2 + (2\pi f_{12})^2}$ is the frequency of the ZQC between the eigenstates $|2\rangle$ and $|3\rangle$. Hence, coherent exchange of polarization is taking place. As $\delta \omega$ becomes smaller the frequency of the oscillations decreases, but the amplitude increases: at $\delta \omega \to 0$ we obtain $\omega_{Q2} = 2\pi f_{12}$ and complete exchange is possible when $t = 1/(2f_{12})$.

In the presence of scalar couplings to the third spin $S$, here a proton ($I_1$ and $I_2$ are carbon-13 nuclei), the expressions should be modified: the evolution should be calculated for each specific spin state of the proton, $|\alpha\rangle$ and $|\beta\rangle$, and sum of the two curves should be taken. We obtain at the following expression:

$$\langle I_{1z}(t) \rangle = 1 - \sin^2 \theta_+ \left( \frac{1 - \cos(\omega^+_Q t)}{4} \right) - \sin^2 \theta_- \left( \frac{1 - \cos(\omega^-_Q t)}{4} \right)$$

$$\langle I_{2z}(t) \rangle = \sin^2 \theta_+ \left( \frac{1 - \cos(\omega^+_Q t)}{4} \right) + \sin^2 \theta_- \left( \frac{1 - \cos(\omega^-_Q t)}{4} \right)$$  \hspace{1cm} (7)

where the evolution frequencies are equal to $\omega_{Q2}^\pm = \sqrt{\delta \omega^2 + (2\pi f_{12})^2}$.

The time dependence of the expectation value for the longitudinal polarizations of spins $I_1$ and $I_2$ is presented in Figure 3 in the presence and the absence of scalar couplings to a heteronucleus. In the absence of heteronuclear coupling the two strongly coupled spins (the strong coupling condition is fulfilled since $2\pi f_{12} > \delta \omega$) almost completely exchange polarizations. The polarization transfer is of a coherent nature and the frequency of the oscillations is close to the scalar-coupling constant $f_{12}$. In the presence of different heteronuclear scalar couplings to the third spin $S$, the time-evolution changes considerably. The two spins are no longer in the regime of strong coupling, since $|\delta \omega| > 2\pi f_{12}$. The efficiency of polarization transfer is reduced and complete exchange of polarization is no longer possible. The time dependence also becomes more complex: instead of a single frequency $\omega_{Q2}$ found in the previous case, here two frequencies appear: $\omega_{Q2}$ and $\omega_{Q2}^\pm$. Hence, when couplings to heteronuclei are present, the condition $\delta \omega \sim 2\pi f_{12}$ does not guarantee that the homonuclei are in the strong-coupling regime.

These results show that the interaction with a heteronucleus clearly alters polarization transfer in strongly coupled networks. Consequently, we expect strong effects of heteronuclear interactions on polarization transfers in systems with several heteronuclei. Notably, we anticipate that polarization transfer among strongly coupled carbon spins will be dramatically different in the presence of proton decoupling, which effectively removes proton-carbon spin-spin interactions.

B. Spin dynamics simulations
In addition to this simple model, we carried out numerical simulations in a realistic multi-spin system: the isopropyl group of carbon-13 labeled leucine. This spin system contains three carbon-13 nuclei \( I_1, I_2, \) and \( I_3: \) the \( C^\prime \) carbon-13 and the two \( C^0 \) carbon-13 nuclei. In addition, the spin system includes seven protons \( S_1: \) each \( C^0 \) nucleus is coupled to the three protons of the methyl group, and the \( C^\prime \) carbon-13 nucleus is coupled to one proton. We model the effects of fast field variation and coherent spin dynamics at low field. We consider two cases, namely, polarization transfer in the presence and in the absence of proton decoupling.

The simulation method is as follows. The semi-selective inversion pulse on spin \( I_2 \) generates the initial density operator for the three-spin \( I \) system:

\[
\sigma_0 = \sigma(t = 0) = I_{1z} + I_{2z} - I_{3z}
\]  

Hence, we generate a population difference for the states \(|\alpha\alpha\beta\rangle, |\alpha\beta\alpha\rangle\) and \(|\beta\alpha\alpha\rangle\): the first state is overpopulated, while the other two states are underpopulated. The three-spin system under study, \( C^0 \), \( C^0 \), and \( C^0 \), has a LAC at \( B = B_{\text{LAC}} \approx 1.1 \text{T} \), see Figure 4. Upon passage through a LAC during the field jump \( B_{\text{HF}} \rightarrow B_{\text{LF}} \) due to the sample shuttle transfer, the population difference is expected to be converted into a coherence between the states, which have the LAC: these adiabatic states correspond to the \(|\alpha\alpha\beta\rangle\) and \(|\alpha\beta\alpha\rangle\) states at high fields. To calculate the actual spin state at \( B = B_{\text{LF}} \) we solve numerically the Liouville-von Neumann equation for the spin density operator

\[
\frac{d}{dt}\sigma = -i[\hat{\mathbf{H}}(t), \sigma]
\]

The Hamiltonian of the spin system at a magnetic field \( B \) is as follows:

\[
\hat{\mathbf{H}}(B) = -\gamma_c B \sum_{i=1}^{3} (1 + \delta_{C_i}) \hat{I}_{1z} - \gamma_h B \sum_{j=1}^{7} (1 + \delta_{H_j}) \hat{S}_{1z} + 2\pi \sum_{i=1}^{3} J_{Ci} (\hat{1}_i \cdot \hat{1}_k) \\
+ 2\pi \sum_{j=m} \sum_{j=m} J_{Hjm} (\hat{S}_j \cdot \hat{S}_m) + 2\pi \sum_{i=1}^{3} \sum_{j=1}^{7} J_{ij}^l \hat{I}_{1z} \hat{S}_{jz}
\]

Here \( \gamma_c \) and \( \gamma_h \) are the carbon and proton gyromagnetic ratios, \( \delta_{C_i} \) and \( \delta_{H_j} \) are the chemical shifts of the \( i \)-th carbon and \( j \)-th proton, \( J_{Cik} \) is the scalar coupling constant between the \( i \)-th and \( k \)-th carbon, \( J_{Hjm} \) is the scalar coupling constant between the \( j \)-th and \( m \)-th proton, \( J_{ij}^l \) is the scalar coupling constant between the \( i \)-th carbon and \( j \)-th proton, \( \hat{1}_i \) and \( \hat{S}_j \) are the spin operator of the \( i \)-th carbon and \( j \)-th proton. Given the range of magnetic fields considered here, heteronuclear scalar couplings are considered to be weak. The precise values of the calculation parameters are given in Table 1. Since the magnetic field \( B \) changes with time, the Hamiltonian \( \hat{\mathbf{H}} \) is also time-dependent. In the calculation we consider three carbons and seven protons (six protons in three \( \text{CH}_3 \)-groups and the \( \gamma \)-proton). Using this Hamiltonian we evaluate the density operator after the first field jump, \( \sigma(t = \tau_1) \). The Liouville-von Neumann equation is integrated using 1 ms time increments and assuming that for each step the Hamiltonian is constant, similarly to simulations carried out for relaxation experiments (Bolik-Coulon et al., 2020). In the calculation, we ignore relaxation effects, since the dimensionality of the relaxation superoperator is too big for the multi-spin system considered here and our focus is on coherent effects.

At \( B = B_{\text{LF}} \) the density operator evolves under a constant Hamiltonian, at the end of the evolution period it becomes as follows:

\[
\sigma(t_1 + \tau) = \exp(-i\hat{\mathbf{H}}(B_{\text{LF}})\tau) \sigma(t_1) \exp(i\hat{\mathbf{H}}(B_{\text{LF}})\tau)
\]  

The \( B_{\text{LF}} \rightarrow B_{\text{HF}} \) field jump is simulated numerically in the same way as the first field jump (the time interval is split into many small steps). Finally, knowing the density operator \( \sigma_{\text{fin}} \) at \( t = \tau_1 + \tau + \tau_2 \), we...
evaluate the NMR signals of the nuclei of interest as the expectation values of their $z$-magnetization.

\[ \langle l_{iz} \rangle = \text{Tr}(l_{iz}\sigma_f) \]

**Table 1.** Parameters used for energy calculations:

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<th>Chemical shifts</th>
<th>Value</th>
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<td>$^1C\delta_2$</td>
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<table>
<thead>
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<th>Value</th>
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</tr>
<tr>
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</tr>
<tr>
<td>$J(C^{\delta_1}, C^{\delta_2})$</td>
<td>0 Hz</td>
</tr>
</tbody>
</table>

**Figure 4.** (A) Energy levels of the $\{C^\gamma, C^{\delta_1}, C^{\delta_2}\}$ spin system at variable magnetic field strength in the absence of scalar coupling with protons. Levels are assigned at high field, where the spin system is weakly coupled. (B) energy levels, corresponding to the $\alpha\alpha\beta$ and $\alpha\beta\alpha$ states at high field, have a LAC at 1.1 T, which is responsible for generation of the zero quantum coherences. To visualize the energy levels better, in the right panels we have subtracted the large Zeeman energy from the actual energy and show the energy difference. The calculation is done using parameters listed in **Table 1** and neglecting carbon-proton couplings.

The method used for modelling the experiments with decoupling at $B = B_{LF}$ is different. After evaluating the density operator $\sigma(t = t_\gamma)$ we trace out the proton degree of freedom and define the density operator of the carbon subsystem as $\sigma_{C}(t_\gamma) = \text{Tr}_{\beta}(\sigma(t_\gamma))$, with the argument that proton polarization is destroyed by decoupling. The partial trace procedure implies that when $\sigma_{k,l}$ is a proton-carbon density operator (in the notation of spin states $i,j$ stand for the proton states and $k,l$ stand for the carbon states), the elements of the carbon density operator are: $[\sigma_{C}]_{k,l} = \sum_i \sigma_{ik,il}$. One should note that proton two-spin operators may contain a zero-quantum component, which would stand proton decoupling. Consideration of effects of such coherences is beyond the scope of this work: we expect this to only lead to small perturbations of the observed behavior. Then we introduce the Hamiltonian of the carbon subsystem

\[ \hat{H}_C(B_{LF}) = -\gamma_C B_{LF} \sum_{i=1}^{3} (1 + \delta_{C_i}) l_{iz} + 2\pi \sum_{i\neq k} J_{Cik}(1_i \cdot 1_k) \]  

(12)
Using this Hamiltonian, we evaluate the density operator of the $^{13}$C spins at the end of the evolution period as follows:

$$\sigma_C(t_1 + \tau) = \exp\left(-i\hat{H}_C(B_{1F})\tau\right) \sigma_C(t_1) \exp\left(i\hat{H}_C(B_{1F})\tau\right)$$  \hspace{1cm} (13)

The final step in evaluating the ZQC evolution is introducing the carbon-proton density operator. This is done by multiplying $\sigma_C(t_1 + \tau)$ and the density operator of non-polarized protons (as decoupling removes any proton spin order). Hence

$$\sigma(t_1 + \tau) = \sigma_C(t_1 + \tau) \otimes \sigma_H^{\text{dec}}, \quad \sigma_H^{\text{dec}} = \frac{1}{2^7} \prod_{j=1}^{7} \mathbb{I}$$  \hspace{1cm} (14)

where $\mathbb{I}$ is a $2 \times 2$ unity matrix. The final step of the calculation, the field jump $B_{1F} \rightarrow B_{HF}$, is modelled in the same way as in the previous case.

Finally, we would like to comment on the $B(t)$ dependence, which was used in calculation. The distance dependence of the magnetic field $B(z)$ is precisely known but the precise $z(t)$ is not known. We modelled this dependence assuming that motion goes with a constant speed (in experiments, constant-speed motion is achieved after a 5-10 ms lag delay for acceleration). Non-ideal agreement between theory and experiment can be attributed to the fact that the precise $z(t)$ dependence is not known (our previous works show that the knowledge of $z(t)$ is required for modeling).

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**IV. Results and discussion**

The experimental $\tau$ dependences of the measured spin polarization are shown in Figure 5. One can see that without decoupling no coherent behavior is found: polarization simply decays due to relaxation and no coherent oscillations are visible (Figure 5A). In the presence of proton decoupling the situation is drastically different: coherent oscillations are clearly observed, which mediate polarization exchange between the $C_{\delta 1}$ and $C_{\delta 2}$ nuclei. We attribute such polarization exchange to the ZQC, which is generated by passage through the LAC. The coherence gives rise to exchange of the populations of the two states, which experience the LAC. These levels are correlated with the $|x\alpha\beta\rangle$ and $|x\beta\alpha\rangle$ high-field states. Hence, polarization transfer gives rise to population exchange of the states $|x\alpha\beta\rangle$ (initially overpopulated state) and $|x\beta\alpha\rangle$ (initially underpopulated state). As a result, the state of the first spin, $C'$, does not change, but the other two spins, $C_{\delta 1}$ and $C_{\delta 2}$, exchange polarizations. With the available speed and range of the field-cycling, other coherences are not excited, i.e., non-adiabatic variation of the Hamiltonian is achieved only for the pairs of levels that have the LAC in between $B_{1F}$ and $B_{HF}$, i.e., only the LAC shown in Figure 4 contributes to spin mixing. The $C'$ spin never shows any oscillatory polarization transfer, which is an...
indication that the specific LAC is responsible for the observed effect. In conclusion, a zero-quantum coherence of the two carbon-13 nuclei $C^{\delta_1}$ and $C^{\delta_2}$ is excited by fast magnetic field jump between 14.1 T and 0.33 T.

The oscillatory behavior does not show up in the absence of proton decoupling. There are two reasons for that. First, the multiple proton-carbon couplings give rise to a set of ZQC frequencies, instead of a unique frequency in the presence of decoupling. Second and more importantly, proton-carbon-13 couplings prevent the carbon subsystem from reaching the strong-coupling regime. Thus, the amplitude of coherent evolutions is drastically reduced (see Eq. 7) and becomes negligible (Fig. 5A). As a result, in experiments without decoupling the ZQC decays because of inhomogeneous broadening of the ZQC evolution frequency, i.e., relaxation. We would like to stress that the ZQC of interest is excited by the field jump, which is identical for experiments with and without proton decoupling at low field. However, the ZQC does not reveal itself and does not give rise to efficient polarization transfer in the experiment without decoupling.

Figure 6. Calculated $r$-dependence of polarization (lines) overlaid with the observed time traces (points) obtained (A) without $^1$H decoupling, and (B) with $^1$H decoupling. The slowly relaxing background (compare with the data shown in Figure 5) has been subtracted from the time traces, to enable comparison between theory and simulations. Observed NMR intensities are normalized to intensities in 150.9 MHz (14.1 T) $^{13}$C spectra at thermal equilibrium. We use the subtraction procedure because relaxation effects were not taken into account in the calculation; consequently, we are unable to consider polarization decay due to relaxation at $B_{LF}$ and during the field variation. To enable comparison of the experiment and calculation results, the amplitude of oscillations in polarization transfer traces were scaled with the same factor, then the starting polarization values were adjusted individually to give best agreement with experimental data.

These considerations are confirmed by theoretical modeling (Figure 6). In the presence of carbon-proton couplings coherent oscillations are hardly observed: only fast oscillations of very small amplitude can be seen in the simulated curves. By contrast, in the absence of the proton-carbon couplings, i.e., when decoupling is used, coherent evolutions become manifest with slower oscillations of larger amplitude. The results of numerical modeling are in good agreement with the experimental data. As relaxation effects are not taken into account in simulations, to ease comparison we subtracted the slowly relaxing background from the experimental time traces. In addition, we rescaled all calculated traces with the same factor; then the starting polarization values were adjusted individually to achieve the best agreement with the experimental data. Such a data treatment becomes necessary because relaxation is active not only during spin mixing at the $B_{LF}$ field, but also during the field jumps. The agreement between the experimental data and simulation in Figure 6 is not ideal, possibly because some small long-range scalar couplings are not included in the simulation but most likely because the field switching profile is not known exactly: previous studies of the spin dynamics in field-cycling NMR experiments (Pravdivtsev et al., 2013; Kiryutin et al., 2013) suggest that using the precise $B(t)$ profile is crucial for simulating coherent polarization transfer phenomena.
The absence of strong-coupling regime, in spite of scalar coupling constants larger than the difference in Larmor frequencies is somewhat counterintuitive but clearly explained when taking into account the effect of large heteronuclear scalar couplings (Eqs. 2-4). In the present case, the effect is even more pronounced since the two carbon-13 nuclei of leucine are coupled to no less than 3 protons each, further splitting resonance frequencies in the absence of proton decoupling. A conventional way to present the weak coupling regime consists in stating that the part of the scalar coupling Hamiltonian (Eq. 1) that is proportional to a zero-quantum product operator is non-secular in the frame of the Zeeman interactions of the two coupled spins, which is true if the scalar coupling constant is much smaller than the difference in Larmor frequencies of the two spins. Here, the weak-coupling regime is extended because this zero-quantum part can be considered non-secular in the interaction frame of the heteronuclear scalar couplings (note that the perturbative treatment is allowed to the extent that the heteronuclear coupling constants are much larger than the homonuclear coupling).

A particular consequence of the observation we report here can be relevant for experiments where the strong scalar-coupling regime is created by radio-frequency irradiation: isotropic mixing for total correlation spectroscopy (TOCSY) (Braunschweiler and Ernst, 1983). We have recently introduced a two-field TOCSY experiment where isotropic mixing is carried out at 0.33 T and chemical shift evolutions occur at high field (Kaděřávek et al., 2017), which makes broadband carbon-13 TOCSY straightforward. This study included a control experiment where no radio-frequency pulses were applied at low field (see Figure 3.b in reference (Kaděřávek et al., 2017). Intuitively, one would have expected cross-peaks to be observed for carbon-13 nuclei in strongly-coupled networks at 0.33 T. Some cross peaks could indeed be observed within the aliphatic carbon region of leucine and in the aromatic ring of phenylalanine. The current investigation suggests that strong scalar couplings between carbon-13 nuclei are less prevalent than expected at 0.33 T. The observed cross-peaks were possibly due to cross relaxation and not necessarily coherent evolution under strong scalar couplings. Conventional TOCSY experiments might also be altered by the effect of large heteronuclear scalar couplings. In this case, isotropic mixing sequences have been optimized on isolated pairs of two coupled spins (Kadkhodaie et al., 1991), excluding the effects of scalar couplings to heteronuclei or as heteronuclear decoupling sequences that happen to be efficient at isotropic mixing (Rucker and Shaka, 1989; Shaka et al., 1988). Although isotropic mixing sequences decouple heteronuclear scalar couplings, optimizing simultaneously for homo- and heteronuclear scalar coupling operators may improve homonuclear coherence transfers.

V. Conclusions

In this work, we present a study of coherent polarization transfer in a system of (strongly) coupled carbon-13 nuclei. Spin coherences are zero-quantum coherences, which are generated by a fast non-adiabatic magnetic field jump. Such coherences are excited most efficiently when the system goes through a LAC during the field switch. Here we indeed pass through a LAC in a system of three coupled carbon-13 spins and investigate the spin dynamics at low fields, where strong couplings of the carbon spins are expected.

We can clearly demonstrate the polarization transfer in the carbon-13 spin subsystem is strongly affected by spin-spin interactions with the protons in the molecule. In this situation, the role of these interactions can be determined by comparing the experiments with and without proton decoupling at low fields. When decoupling is used, we observe coherent polarization exchange between two of the three carbons: such a behavior is typical when the spin coherences are excited upon non-adiabatic passage through a specific LAC. In the absence of decoupling, i.e., when heteronuclear interactions are present, we cannot observe such a behavior: polarization transfer is very inefficient and coherent phenomena are not found. We attribute this to the fact that relatively strong proton-carbon couplings (i) drive the carbon system away from the strong coupling condition and (ii) give rise to a set of evolution frequencies instead of a unique ZQC frequency. These considerations are supported by an analytical model of a three-spin system and numerical simulations in a multi-spin system.
Our results are of importance for analyzing polarization transfer phenomena at low magnetic fields and for interpreting NMR data obtained under apparent strong coupling conditions. Under such conditions heteronuclear spin-spin interactions might disturb “strong coupling” of homonuclei and substantially alter spin dynamics.

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