Heteronuclear and Homonuclear Finite Pulse Radio Frequency Driven Recoupling

Authors: Evgeny Nimerovsky*, Kai Xue, Kumar Tekwani Movellan & Loren B. Andreas*

Affiliations:
Department of NMR based Structural Biology, Max Planck Institute for Biophysical Chemistry, Am Fassberg 11, Göttingen, Germany

*Corresponding authors: land@nmr.mpibpc.mpg.de ORCID: 0000-0003-3216-9065 and evni@nmr.mpibpc.mpg.de

Abstract
Homonuclear finite-pulse radio frequency driven recoupling (fp-RFDR) has been broadly used in multi-dimensional magic-angle spinning (MAS) solid-state NMR experiments over the past 20 years. The theoretical and the simulated descriptions of this method were presented during that time, resulting in an understanding of the influence of chemical shift offset, finite pulse effects, and dipolar truncation. Here we present an operator analysis of both heteronuclear and homonuclear fp-RFDR. By numerical simulation, we show which operators are involved in the longitudinal exchange for both heteronuclear and the well-known homonuclear sequences. This results in a better understanding of the influence of phase cycling of the fp-RFDR pulses, which is typically a variant of XY cycling. We investigate the heteronuclear and homonuclear fp-RFDR signals and evolution of the operators through the fp-RFDR block. We show the convergence of the evolutions of the heteronuclear and homonuclear fp-RFDR signals at even numbers of rotor periods and completely different evolution between them. We demonstrate heteronuclear $^1$H- $^{13}$C and $^1$H-$^{15}$N fp-RFDR magnetization transfer using a microcrystalline SH3 sample at 100 kHz MAS.
Introduction

Solid-state magic-angle spinning NMR spectroscopy is used to obtain atomic resolution physical and chemical knowledge about the investigated sample. One of the abilities of the NMR experiments is to determine the distance between a pair of spin 1/2 nuclei via recoupling the homonuclear (A. E. Bennett et al. 1992; Ok et al. 1992; Z. Zhang et al. 2020; Gelernter, Dregni, and Hong 2020; Takegoshi, Nakamura, and Terao 2001; Szeverenyi, Sullivan, and Maciel 1982; Hou, Yan, et al. 2011; Hou et al. 2013; Carravetta et al. 2000; Andrew E. Bennett et al. 1998) or heteronuclear (Gelernter, Dregni, and Hong 2020; T. Gullion and Schaefer 1989; Jaroniec, Filip, and Griffin 2002; Hing, Vega, and Schaefer 1992; Hartmann and Hahn 1962; Rovnyak 2008; Metz, Wu, and Smith 1994; Hediger et al. 1994; Hou, Byeon, et al. 2011; Brinkmann and Levitt 2001; Gelernter and Hong 2018; Z. Zhang, Chen, and Yang 2016) dipolar interactions. The homonuclear fp-RFDR sequence is successfully applied for the qualitative and quantitative determinations of the dipolar spin correlations in materials (Saalwächter 2013; Messinger et al. 2015; Fritz et al. 2019; Roos, Mandala, and Hong 2018; Nishiyama et al. 2014; Wong et al. 2020; Hellwagner et al. 2018; Pandey and Nishiyama 2018) and biomolecular samples (Zheng, Qiang, and Weliky 2007; Tang, Berthold, and Rienstra 2011; Shen et al. 2012; Pandey et al. 2014; Grohe et al. 2019; Andreas et al. 2015; Petkova et al. 2002; Aucoin et al. 2009; Zinke et al. 2018; R. Zhang, Mroue, and Ramamoorthy 2017; Zhou et al. 2012; Jain et al. 2017; Colvin et al. 2015; Shi et al. 2015; Daskalov et al. 2020).

These applications depend on a firm quantum mechanical foundation. One of the theoretical tools to investigate the influence of radio frequency (RF) pulse sequences on the spin system is Average Hamiltonian Theory (Haeberlen and Waugh 1968; Maricq 1982) (AHT). The two necessary conditions for application of the AHT are (Ernst, Bodenhausen, and Wokaun 1987):

1. The total Hamiltonian has to be periodic;
2. The stroboscopic measurements are synchronized with the period, or cycle time, of the total Hamiltonian.

When these conditions are fulfilled, the time-dependent Hamiltonian, evaluated at multiples of the cycle time, can be replaced by the sum of the time-independent multiple order averaging terms (Ernst, Bodenhausen, and Wokaun 1987).

AHT simplifies quantum calculations, especially in cases when complex multiple-pulse sequences are used. It can explain the selectivity of the pulse sequence, meaning to find the experimental conditions under which the desired interactions are recoupled and undesired decoupled. However, AHT can predict the state of the spin system at specific time points only and not the paths by which the spin system evolves during the period when rf pulses are given. Another successful method, Floquet Theory (Levante et al. 1995; Scholz, van Beek, and Ernst 2010), allows to consider the Hamiltonian at any point of time. However, such analysis is complicated with a transformation to infinity-dimensional Hilbert space (Levante et al. 1995).

Homonuclear transfer of the magnetization via longitudinal exchange occurs with a rotor-synchronized train of π-pulses, with one pulse each rotor period. The method is called radio-frequency driven recoupling (A. E. Bennett et al. 1992) (RFDR), or simple excitation for the dephasing of rotational-echo amplitudes (Terry Gullion and Vega 1992). This sequence has two different AHT descriptions of the recoupling of homonuclear dipolar interactions, depending on the experimental conditions (Ok et al. 1992; Ishii 2001).

In the first case, delta π-pulses are assumed. The efficiency to recouple homonuclear dipolar interaction is linked with the difference between isotropic chemical shifts of the dipolar linked spins, $S_k$ and $S_l$ (A. E. Bennett et al. 1992; Terry Gullion and Vega 1992; Andrew E. Bennett et al. 1998). The evolution of the spin system at specific time points is described with a flip-flop part of zero-quantum

https://doi.org/10.5194/mr-2020-30
Preprint. Discussion started: 20 November 2020
© Author(s) 2020. CC BY 4.0 License.

For the second theoretical description, finite $\pi$-pulses are considered (fpRFDR)(A. E. Bennett et al. 1992; Ishii 2001; Nishiyama, Zhang, and Ramamoorthy 2014; R. Zhang et al. 2015; Brinkmann, Schmedt auf der Günne, and Levitt 2002; Ji et al. 2020). The efficiency of recoupling of the homonuclear dipolar interaction is directly linked with a duty factor(Ishii 2001) – the ratio between the width of $\pi$-pulse and the width of the rotor period. AHT predicts restoring of the whole zero-quantum dipolar Hamiltonian, $3I_k^zI_l^z - \bar{I}_k\bar{I}_l$, under fast and ultra-fast MAS rates(Ishii 2001).

Both these theoretical descriptions consider the same experiment with the same phase cycling, traditionally XY8 (Terry Gullion, Baker, and Conradi 1990). Although the influence of the different phase cycling schemes was investigated in different articles(Ok et al. 1992; Nishiyama, Zhang, and Ramamoorthy 2014; R. Zhang et al. 2015; Ji et al. 2020), the main conclusion to the contribution from phase cycling to the transfer of the RFDR signal was a reduction of influence from resonance offsets and pulse errors(A. E. Bennett et al. 1992; Ishii 2001).

In this article we investigate both heteronuclear and homonuclear fpRFDR experiments using numerical tools to track the system at any arbitrary time. Using the simulated and the theoretical analysis we show that for fpRFDR experiments the typical phase cycling, XY(Ishii 2001; Nishiyama, Zhang, and Ramamoorthy 2014; R. Zhang et al. 2015; Hellwagner et al. 2018), plays a crucial role in the transfer of magnetization between a pair of spins. Under fast and ultra-fast MAS rates the heteronuclear and homonuclear fpRFDR experiments can be described with the same model Hamiltonian, but only at increments of the rotor period. The evolutions of the operators, however, are completely different for heteronuclear and homonuclear cases between these points. For the experimental demonstrations we perform heteronuclear 1D $^1H-{^13}C$ and $^1H-{^15}N$ fp-RFDR experiments using $\alpha$-PET (Movellan et al. 2019) labeled SH3 at 100 kHz MAS.
Theory

The fp-RFDR sequence consists of a train of $\pi$-pulses every one rotor period (Fig.1a). The length of the repeated block is defined by the phase cycling: $XYn$ ($n=4, 8, 16, 32$), resulting in a time of $nT_R$ (Ishii 2001). Measurements are, in the simplest case, restricted to occur every $nT_R$. In our simulations as well as in the experiments we used $XY8$ phase cycling.

To evaluate the operator, $\hat{A}$, between the time points $t_i$ and $t_i + t_x$ we first have to solve the von Neumann equation ($\hbar = 1$):

$$\frac{d\rho}{dt} = -i[H(t), \rho(t)], \quad \text{Eq. (1.1)}$$

where $H(t)$ is a Hamiltonian of the spin system and $\rho(t)$ is a density matrix. The formal solution of the Eq. (1.1) (Ernst, Bodenhausen, and Wokaun 1987) is:

$$\rho(t_i + t_x) = \hat{T}\exp\left(-i \int_{t_i}^{t_i + t_x} dt \, H(t)\right)\rho(t_i)\hat{T}\exp\left(-i \int_{t_i}^{t_i + t_x} dt \, H(t)\right), \quad \text{Eq. (1.2)}$$

where $\hat{T}$ is a Dyson Operator.

The evaluated operator, $\hat{A}(t_i + t_x)$ is:

$$\hat{A}(t_i + t_x) = Tr\{\hat{A}\rho(t_i + t_x)\} \quad \text{Eq. (1.3)}$$

One of the possibilities to deal with a Dyson Operator in Eq. (1.2), in order to propagate forward in time from the point $t_i$ to $t_i + t_x$, is to split one propagator into a product of $N$ propagators(Nimerovsky and Goldbourt 2012):

$$\hat{T}\exp\left(-i \int_{t_i}^{t_i + t_x} dt \, H(t)\right) = \lim_{N \to \infty} \prod_{k=1}^{N} \exp\left(-i \int_{t_i + k\Delta x_{i,k}}^{t_i + (k+1)\Delta x_{i,k}} dt \, H(t)\right), \quad \Delta x_{i,k} = t_x/N \quad \text{Eq. (1.4)}.$$

It allows to omit the Dyson Operator and perform the simulations correctly. This is the way that calculations are performed in the popular SIMPSON software(Bak, Rasmussen, and Nielsen 2000). The main difference in our implementation of numerical evolutions with respect to SIMSPON calculations is

https://doi.org/10.5194/mr-2020-30
Preprint. Discussion started: 20 November 2020
© Author(s) 2020. CC BY 4.0 License.
to use analytical integrations rather than numerical integrations, which significantly reduces the computer
time.

Each of the rotor periods of the fp-RFDR sequence can be divided into two parts with the lengths
of $t_p$ (defined with the length of $\pi$-pulse and $T_R - t_p$ (the delay, Fig. 1a). The numerical calculations split
each of these two parts into $N$ subparts with the lengths $t_p/N$ and $(T_R - t_p)/N$, respectively. Fig. 1b shows
the transferred fp-RFDR signals for $I_3$ spin system under different values of $N$. Solid lines represent the
transferred $I_2 \rightarrow I_3$ signal between a weakly coupled dipolar pair with 66 Hz dipolar interaction, whereas
the dotted lines represent the $I_2 \rightarrow I_2$ signal. With an increase in the value of $N$, the simulated signals
converge and under $N=16$ (red lines) and $N=32$ (black lines) the signals coincide. It means that under
$N \geq 16$, the simulations provide the correct evolution of the spin system. In all numerical calculations we
used $N=32$.

Fig. 1 (a) Finite Pulse RFDR block consisting of four $\pi$-pulses every rotor period with phase cycling: $x$, $y$, $x$, $y$ (XY4) or eight $\pi$-
pulses with phase cycling: $x$, $y$, $x$, $y$, $x$, $y$, $x$ (XY8). (b) Simulated $I_2$ spin system under 10 kHz of MAS and 65 kHz of rf-field,
dipolar values of $\nu_{12} = 66$ Hz, $\nu_{13} = 150$ Hz, $\nu_{23} = 2.15$ kHz, and offset values of $\Omega_1 = -8$ kHz, $\Omega_2 = 9$ kHz, $\Omega_3 = -7$ kHz
and CSA values of $\nu_{C\text{SA}1} = 9.2$ Hz, $\nu_{C\text{SA}2} = 2.5$ Hz, $\nu_{C\text{SA}3} = 8$ kHz. Axis Y shows the intensities of the starting and
transferred signals between different operators, $I_{22} \rightarrow I_{22}$ (the dotted lines) and $I_{22} \rightarrow I_{21}$ (the solid lines), under different values
of $N$: 1 (magenta lines), 2 (green lines), 8 (blue lines), 16 (red lines) and 32 (black lines). The values of the dipolar interactions
and MAS rate were taken from Ref. [(Bayro et al. 2009)]. XY8 phase cycling was used.
Figs. A1-2 show the transferred fp-RFDR signals for $I_3$ and $IS_2$ spin systems under different simulated conditions and validation by comparison with SIMSPON simulations of fp-RFDR signals.

In the next ‘Simulations’ section we firstly consider the evolution of the $I_3$ spin system under the RFDR sequence. We investigate the influence of each part of the Dipolar Hamiltonian (the secular and the flip-flop parts) on the measured operators at specific time points under different simulated conditions.

In the subsequent subsections we compare the behavior of $I_2$ and $IS$ spin systems under RFDR. We consider the evolutions of all operators and their amplitude at different time points. We demonstrate with simulations and provide the theoretical analysis (applying the fictitious spin $\frac{1}{2}$ operator formalism(Vega 1978)) of the influence of the phase cycling on the transfer of fp-RFDR signal. We shows the paths with which the signals are transferred between different spins for heteronuclear and homonuclear spin systems.

Simulations

The full high field truncated dipolar Hamiltonian of $I_3$ spin system is represented as follows:

$$H_{D,Full} = 0.5 \sum \omega_{D,rs}(t) [3I_{zr}I_{zs} - I_rI_s] = 0.5 \sum \omega_{D,rs}(t) [3I_{zr}I_{zs} - I_{zr}I_{zs} - 0.5(I^+_rI^-_s + I^+_rI^+_s)] , \text{ Eq. (2.1)}$$

where $\omega_{D,rs}(t)$ is a periodic dipolar time dependent function(Olejniczak, Vega, and Griffin 1984) between spins $I_r$ and $I_s$. This Hamiltonian is subsequently referred to as the full Hamiltonian, and contains only the A and B terms of the dipolar alphabet(Slichter 1990).

Firstly we investigate which part of the full high field dipolar Hamiltonian can be a model Hamiltonian. A model Hamiltonian is a simplified Hamiltonian, which provides the same evolution of the spin system at specific time points as a full dipolar Hamiltonian. We consider 3 model Hamiltonians:

$$H_D = 1.5 \sum \omega_{D,rs}(t)I_{zr}I_{zs} , \text{ Eq. (2.2a)} \quad H_D = -0.5 \sum \omega_{D,rs}(t)I_rI_s , \text{ Eq. (2.2b)}$$

$$H_D = -0.25 \sum \omega_{D,rs}(t)[I^+_rI^-_s + I^-_rI^+_s] , \text{ Eq. (2.2c)}$$
Eq. (2.2a) contains a secular part of the dipolar Hamiltonian (Eq. (2.1)), whereas the scalar products, \( I_s I_s \), are omitted. Eq. (2.2b) contains the scalar products of the dipolar Hamiltonian only. Eq. (2.2c) contains the flip-flop parts of the Dipolar Hamiltonian. The Figs. 2-4 show the evolution of a \( I_3 \) spin system under these three model Hamiltonians (Eq. (2.2a) – red lines; Eq. (2.2b) – green lines; Eq. (2.2c) – pink lines) and comparison with the full Dipolar Hamiltonian (Eq. 2.1 – black lines).

Fig. 2 shows the transferred signals under 10 kHz of MAS, when only dipolar interactions are taken into account. Under these conditions the secular dipolar Hamiltonian (Fig. 2a, red lines) provides similar values of the amplitudes of the operators at specific time points (the simulated measurement occurred every \( 8T_R \), with XY8 phase cycling, as with the full Dipolar Hamiltonian (Fig. 2a, black lines). The scalar product dipolar Hamiltonian (Fig. 2b, green lines) and the flip-flop dipolar Hamiltonian (Fig. 2b, pink lines) provide different results with respect to the full Dipolar Hamiltonian (Fig. 2b, black lines).

**Fig. 2** Simulated \( I_3 \) spin system under 10 kHz of MAS and 65 kHz of rf-field, dipolar coupling values of \( \nu_{12} = 66 \text{ Hz}, \nu_{13} = 150 \text{ Hz}, \nu_{23} = 2.15 \text{ kHz} \) (the schematic spin arrangement is shown in the insert of (b)) and zero values of offset and CSA. Axis Y shows the intensities of the starting and transferred signals between different operators: \( I_2 \rightarrow I_2 \) (the dotted lines); \( I_2 \rightarrow I_3 \) (the dashed lines); \( I_3 \rightarrow I_2 \) (the solid lines); \( I_3 \rightarrow I_3 \) (the dashed-dotted lines). (a) The black lines represent the signals, simulating with the full dipolar Hamiltonian (Eq. 2.1). The red lines represent the signals, simulating with the secular model Hamiltonian (Eq. 2.2a). (b) The black lines represent the signals, simulating with the full dipolar Hamiltonian (Eq. 2.1). The green lines represent the signals, simulating with the scalar product model Hamiltonian (Eq. 2.2b). The magenta lines represent the signals,
simulating with the scalar product model Hamiltonian (Eq. 2.2c). The values of the dipolar interactions and MAS rate was taken from Ref. [(Bayro et al. 2009)]. XY8 phase cycling was used.

The conclusions are reversed when we add chemical shift offsets that have similar values with respect to the MAS frequency (Fig. 3). In the simulations CSA values were also added into simulations. However, the main influence comes from the offset as shown in Fig. (A2). The secular dipolar Hamiltonian (Fig. 3a, red lines) provides completely different results with respect to the full dipolar Hamiltonian (Fig. 3a, black lines), whereas the scalar product dipolar Hamiltonian (Fig. 3b, green lines) and the flip-flop dipolar Hamiltonian (Fig. 3b, pink lines) provide similar results as the full dipolar Hamiltonian (Fig. 3b, black lines).

**Fig. 3** Simulated $I_3$ spin system under 10 kHz of MAS and 65 kHz of rf-field, dipolar coupling values of $\nu_{12} = 66$ Hz, $\nu_{13} =$ 150 Hz, $\nu_{23} = 2.15$ kHz, offset values of $\Omega_1 = -8$ kHz, $\Omega_2 = 9$ kHz, $\Omega_3 = -7$ kHz, and CSA values of $\nu_{\text{CSA,1}} =$ 9.2 Hz, $\nu_{\text{CSA,2}} = 2.5$ Hz, $\nu_{\text{CSA,3}} = 8$ kHz. Axis Y shows the intensities of the starting and transferred signals between different operators: $I_{22} \rightarrow I_{22}$ (the dotted lines); $I_{22} \rightarrow I_{23}$ (the dashed lines); $I_{22} \rightarrow I_{21}$ (the solid lines); $I_{23} \rightarrow I_{21}$ (the dashed-dotted lines).

(a) The black lines represent the signals, simulating with the full dipolar Hamiltonian (Eq. 2.1). The red lines represent the signals, simulating with the secular model Hamiltonian (Eq. 2.2a). (b) The black lines represent the signals, simulating with the full dipolar Hamiltonian (Eq. 2.1). The green lines represent the signals, simulating with the scalar product model Hamiltonian (Eq. 2.2b). The magenta lines represent the signals, simulating with the scalar product model Hamiltonian (Eq. 2.2c). The values of the dipolar interactions and MAS rate were taken from Ref. [(Bayro et al. 2009)]. XY8 phase cycling was used.
When the spinning frequency is increased such that it is significantly larger than the offsets, the situation again reverses. Fig. 4 shows the evolution of the spin system under 90 kHz MAS with the same offset and CSA values as in Fig. 3. Under such conditions the secular dipolar Hamiltonian (Fig. 4a, red lines) provides similar results as a full dipolar Hamiltonian (Fig. 3a, black lines), whereas the scalar product dipolar Hamiltonian (Fig. 4b, green lines) and the flip-flop dipolar Hamiltonian (Fig. 4b, pink lines) provide different results with respect to full dipolar Hamiltonian (Fig. 4b, black lines).

Considering these three cases, we can conclude that for fp-RFDR, when the difference of the offset values between spins are significantly smaller with respect to the used MAS rate, the simplified secular Hamiltonian (Eq. 2.1a) can play a significant role in the transfer. Such a simplified model...
Hamiltonian correctly describes the evolution of the spin systems at specific time points. In the case when offset values are comparable with the MAS rate, the flip-flop Hamiltonian (Eq. 2.1c) can play a role of the simplified model Hamiltonian. The second conclusion coincides with the theoretical analysis previously obtained with AHT for that sequence (A. E. Bennett et al. 1992; Andrew E. Bennett et al. 1998; Nielsen et al. 1994; Ok et al. 1992; Bayro et al. 2009; Sodickson et al. 1993; Straasø et al. 2016), whereas the first conclusion diverges – dependence on the zero-quantum dipolar operator (Ishii 2001) for AHT. The second conclusion allows the transfer of the magnetization between homonuclear spins only, whereas the first conclusion supposes that a heteronuclear fp-RFDR transfer is also possible.

For the heteronuclear spin system, the full high field dipolar Hamiltonian is secular. The main difference to the heteronuclear full dipolar Hamiltonian with a homonuclear model Hamiltonian (Eq. 2.1a) is a factor of 1.5. On the basis of the first conclusion, for fully heteronuclear spin systems all dipolar interactions should be 1.5 times larger with respect to the homonuclear dipolar values to obtain the same signals as for a fully homonuclear spin system. For a more comprehensive investigation of the behaviors of the homonuclear and heteronuclear spin systems under fpRFDR, we consider below $I_2$ and $I_3$ spin systems.

The simulations allow us to consider the evolutions within a Cartesian operator basis set of the two spin system (Ernst, Bodenhausen, and Wokaun 1987). For the two spin system, the basis set consists of 16 operators. The evolution of each of them is described by microscopic amplitude:

$$a_k(t_{mix}) = Tr \{ \hat{R} \rho(t_{mix}) \}, \quad \text{Eq. (2.3)}$$

where $\hat{R}$ is an operator of the spin system. The macroscopic amplitude is represented as follow:

$$A_k = \int d\Omega a_k(t_{mix}), \quad \text{Eq. (2.4)}$$

where the integration is performed over all Euler angles. Summing all amplitudes in squares, the next condition should be performed:
\[ \sum |a_k|^2 (t_{mix}) = 1 \quad \text{Eq. (2.5)} \]

Fig. 5 shows the macroscopic amplitudes (Eq. 2.4) of 14 operators under different offset values for IS (a, c, e, g) and I2 (b, d, f, h) spin systems. In all Figs. the initial operator is \( I_z \) for the heteronuclear case and \( I_z1 \) for the homonuclear spin systems and the MAS rate is 10 kHz. The pink dashed lines represent the sum of the square amplitudes (Eq. (2.5)).

For the on-resonance condition (Fig. 5a) the evolution of the IS spin system is described with four operators only: \( I_z \) (black line), \( S_z \) (green line), \( 2I_zS_y \) (blue line) and \( 2I_zS_x \) (red line). The pink dashed line shows the sum of the squares of amplitudes of these four operators. The other 11 operators are not excited and have zero amplitudes. For an I2 spin system (Fig. 5b) the evolution is identical as for an IS spin system, when the measurements are taken every two rotor periods.

With a 3 kHz offset difference between the spins (Fig. 5c), we find the excitation of all 14 operators. However, only six of them have significant amplitudes. Additionally to the previously mentioned four operators, we see significant amplitudes for \( 2I_zS_x \) (cyan line) and \( 2I_zS_y \) (purple line) operators. Compared to the on-resonance case, the velocity of the transfer of the signal from \( I_z \) to \( S_z \) operators (green line) decreased for the IS spin system (Fig. 5c). For the I2 spin system (Fig. 5d) the velocity of the transfer of the signal from \( I_{z1} \) to \( I_{z2} \) operators (green line) as well as the velocities of the evolutions of other operators significantly increased.

For a 5 kHz offset difference and heteronuclear IS spins (Fig. 5e), we do not see any transfer of the signal from \( I_z \) operator to \( S_z \) operator, whereas for the I2 spin system (Fig. 5f), the velocities of the excitation of all operators increased more as compared to the previous cases. Also for this case only the 6 previously mentioned operators have significant macroscopic amplitudes.

Under a 9 kHz offset difference and heteronuclear IS spins (Fig. 5g), we see again the transfer of the signal from \( I_z \) operator to \( S_z \) operator (Fig. 5g, green line). Also more operators have significant macroscopic amplitudes. For the I2 spin system (Fig. 5h), the transfer of signal from \( I_{z1} \) to \( I_{z2} \) operators
(green line) has the largest velocity with respect to previous cases. Also here we see more transfer of the signal from $I_{z1}$ to other operators, however with smaller amplitudes in comparison with the IS case.

For negligible offset differences with respect to the MAS rate, the evolution of the operators of IS and $I_2$ spin systems are the same at specific time points. However, when offsets are comparable with the MAS rate, we obtain completely different evolution for these systems. With increase of offset difference the IS spin system passes through specific rotor resonance condition (the difference between offsets equal to half of the MAS rate), under which the transfer does not occur. For the $I_2$ spin system the velocity of the transfer increases with increased offset difference.
Fig. 5 The simulated amplitudes of the operators for IS ((a), (c), (e), (g) – $\nu_{D,IS} = 3$ kHz, the initial operator is $I_x$ and $I_z$ ((b), (d), (f), (h) – $\nu_{D,IS} = 2$ kHz, the initial operator is $I_x$) spin systems under 10 kHz of MAS rate and 65 kHz of rf-field. The pink dashed line represent the sum of the squared amplitudes (Eq. (2.5)), which are shown in the Figs. Black lines – $I_x$ and $I_z$; Green lines – $S_x$ and $I_z$; Blue lines – $2I_xS_x$ and $2I_zI_z$; Red lines – $2I_xS_z$ and $2I_zI_z$; Cyan lines – $2I_xS_x$ and $2I_zI_z$; Purple lines – $2I_xS_y$ and $2I_zI_z$; Azure lines – $S_x$ and $I_z$; Dark green lines – $S_y$ and $I_z$; Orange lines – $I_x$ and $I_z$; Grey lines – $I_x$ and $I_z$; Crimson red lines – $2I_xS_x$ and $2I_zI_z$; Brown lines – $2I_xS_x$ and $2I_zI_z$; Jazzberry Jam lines – $2I_xS_y$ and $2I_zI_z$; Indigo lines – $2I_xS_z$ and $2I_zI_z$. (a) and (b) – Offset values in kHz: 0, 0; CSA values in kHz: 0, 0. (c) and (d) – Offset values in kHz: 2, -1; CSA values in kHz: 9.2, 2.5. (e) and (f) – Offset values in kHz: 2, -3; CSA values in kHz: 9.2, 2.5. (g) and (h) – Offset values in kHz: 2, -7; CSA values in kHz: 9.2, 2.5.

Figs. 5a and b showed the identical evolutions of the same operators for IS and $I_2$ spin system at specific time points (every two rotor periods). A more interesting case is a comparison of the evolution of these operators between specific time points, between 0 and $2T_R$. Fig. 6 shows the microscopic amplitudes (Eq. 2.4) of the operators during first two rotor periods. The time scale of that two rotor periods can be divided into four parts: $t(\pi_x) \rightarrow del_1 \rightarrow t(\pi_y) \rightarrow del_2$. For increasing the effect of the transfer we simulated with 15 kHz and 10 kHz for the dipolar interactions for IS and $I_2$ spin systems, respectively.

Regardless the offset values, the evolutions of the operators between specific time points are completely different for IS (Fig. 6a and c) and $I_2$ spin systems (Fig. 6b and d). For the on-resonance condition, by the end of the first $\pi_x$-pulse only one operator is created for the IS spin system: $2I_xS_x$ (Fig. 6a, blue line). During the first delay, $del_1$, there is no evolution of the spin system since $[I_x, I_yS_y] = [I_x, S_yI_y] = 0$. At the end of the second $\pi_y$-pulse two additional operators have nonzero amplitude: $2I_xS_x$ (Fig. 6a, red line) and $S_z$ (Fig. 6a, green line). During the second delay, $del_2$, the amplitudes of these operators are not changed.

Under on-resonance conditions for the $I_2$ spin system (Fig. 6b) all these four operators have nonzero amplitudes in the end of the first $\pi_x$-pulse. The evolution of the $I_2$ spin system during this two rotor periods is much more complicated as compared to the IS spin system: the amplitudes of these four operators are changed during the delay times as well. However, at the end of two rotor periods the
amplitudes of similar operators of IS and I2 spin systems – $I_z$ and $I_{z1}$; $S_z$ and $I_{z2}$; $2I_xS_y$ and $2I_{x1}I_{y2}$; $2I_yS_x$ and $2I_{y1}I_{x2}$ – have the same values.

For a 9 kHz offset difference, in the end of the first $\pi_x$-pulse only one operator is created for an IS spin system: $2I_xS_y$ (Fig. 6c, blue line). However, during the first delay, $d_{el_1}$, three operators are created: $2I_xS_y$ (Fig. 6c, red line), $2I_yS_x$ (Fig. 6c, cyan line) and $2I_yS_x$ (Fig. 6c, purple line). In the end of the second $\pi_y$-pulse, the operator $S_z$ (Fig. 6c, green line) has nonzero value. However it has a very small value in comparison with the on resonance case (Fig. 6a).

For a 9 kHz offset difference, the evolution of the I2 spin system is also described with six operators (Fig. 6d). However, in that case the $I_{z2}$ operator has much larger amplitude in the end of two rotor periods as compared with the on resonance condition (Fig. 6b).

**Fig. 6** The simulated amplitudes of the operators of a single crystal (Euler angles: $184^\circ$; $141^\circ$; $349^\circ$) during first rotor periods for IS ((a), (c)) – $v_{D,IS} = 15$ kHz, the initial operator is $I_z$ and I2 ((b), (d)) – $v_{D,I2} = 10$ kHz, the initial operator is $I_{z1}$ spin systems under 10 kHz of MAS and 65 kHz of rf-field. The pink dashed line represents the sum of the squared amplitudes (Eq. (2.5)).
which are shown in Figs. Black lines – $I_z$ and $I_{z1}$; Green lines – $S_z$ and $I_{z2}$; Blue lines – $2I_xS_y$ and $2I_1I_y2$; Red lines – $2I_yS_x$ and $2I_y1I_x2$; Cyan lines – $2I_yS_x$ and $2I_y1I_x2$; Purple lines – $2I_xS_y$ and $2I_x1I_y2$. (a) and (b) – Offset values in kHz: 0, 0; CSA values in kHz: 0, 0. (c) and (d) – Offset values in kHz: 2, -7; CSA values in kHz: 9.2, 2.5. The XY phase cycling was used.

Figs. 6a and b (heteronuclear and homonuclear, respectively) show completely different behavior for the amplitudes of four operators – $I_z$, $S_z$, $2I_xS_y$, $2I_yS_x$ for IS and $I_{z1}$, $I_{z2}$, $2I_x1I_y2$, $2I_y1I_x2$ for $I_2$ spin systems during two rotor periods. However, by the end of two rotor periods, the values are the same again. For an $I_2$ spin system the signal is transferred to the $I_{z2}$ operator gradually during two rotor periods (Fig. 6b green line). For IS spin system the signal from $I_z$ to $S_z$ is transferred during the second $\pi_y$-pulse only. Therefore, the phase of the second $\pi$-pulse is also the object for investigation. We considered the behavior of the amplitudes of the operators during two rotor periods when both these pulses had the same phase – $t(\pi_x) \rightarrow \text{del}_1 \rightarrow t(\pi_x) \rightarrow \text{del}_2$. Fig. 7 shows the amplitudes of the operators for IS (Fig. 7a) and $I_2$ (Fig. 7b) spin systems. For an IS spin system (Fig. 7a) only two operators have nonzero amplitudes during the investigated time: $I_z$ (black line) and $2I_xS_y$ (blue line), whereas $S_z$ and $2I_yS_x$ are not created. For the $I_2$ spin system (Fig. 7b) we still detect the evaluations of all four operators. However, in the end of two rotor periods only two operators have nonzero amplitudes, as for the IS spin system. For both cases, there is no transfer of signal from $I_z$ to $S_z$ (IS spin system) and from $I_{z1}$ to $I_{z2}$ ($I_2$ spin system), when XX phase cycling is used. Definitely, the phase cycling of fp-RFDR sequence plays a crucial role in the transfer of the signal between different spins.
Fig. 7 The simulated amplitudes of the operators of a single crystal (the used Euler angles: 184°; 141°; 349°) during first rotor periods for IS ((a) – \nu_{D,IS} = 15 \text{ kHz}, the initial operator is } I_z) and \( I_y \) ((b) – \nu_{D,II} = 10 \text{ kHz}, the initial operator is } I_z) spin systems under 10 kHz of MAS and 65 kHz of rf-field. The XX phase cycling was used. The pink dashed line represents the sum of the squared amplitudes (Eq. (2.5)), which are shown in Figs. Black lines – \( I_x \) and \( I_z \); Green lines – \( S_x \) and \( I_z \); Blue lines – 2\( I_x S_y \) and 2\( I_y I_z \); Red lines – 2\( I_y S_x \) and 2\( I_y I_z \); Cyan lines – 2\( I_z S_x \) and 2\( I_y I_z \); Purple lines – 2\( I_z S_y \) and 2\( I_y I_z \). Offset values in kHz: 0, 0; CSA values in kHz: 0, 0.

To understand the evolution of the IS operators under different phase cycling, we can consider their evolutions during the second \( \pi \)-pulse only (Fig. 8). Fig. 8a shows the amplitudes during a \( \pi \)-pulse with \( y \) phase. During this pulse two additional operators are created. The signal is transferred from operator 2\( I_x S_y \) to \( S_x \) (Fig. 8a, green line) and from \( I_y \) to \( I_z S_x \) (Fig. 8b, red dashed line). During the first pulse with \( x \) phase, 2\( I_x S_y \) is created, whereas during the second pulse with 90° phase shifting the signal is transferred from that operator to \( S_x \). It means that if we consider the evolution of the operators during a third \( \pi \)-pulse with the \( x \) phase, the transfer of the signal from \( I_y \) to operator \( S_x \) will be via the operator 2\( I_x S_y \). When the second \( \pi \)-pulse has the same phase as a first (XX), operators 2\( I_x S_y \) and \( S_x \) are not created and the transfer of the signal from \( I_y \) to \( S_x \) does not occur. In Appendix B we show the formal proof of zero transfer signal from \( I_{z1} \) to \( I_{z2} \) at specific time points (every one rotor period) when XX phase cycling is used.
Fig. 8 Simulated amplitudes of the operators of an IS spin system ($\nu_{DS} = 15$ kHz) for a single crystal (Euler angles: 184°; 141°; 349°) during the second $\pi$-pulse with phase $\gamma$ (a) and $x$ (b) under 10 kHz of MAS and with a 65 kHz rf-field. The signal is shown as a function of the pulse flip angle $\alpha$. The transfer of the signal between: $2I_xS_y \rightarrow I_x$ – green solid lines; $2I_xS_y \rightarrow I_x$ – black solid lines; $I_x \rightarrow I_x$ – black dashed lines; $2I_xS_y \rightarrow 2I_xS_y$ – blue solid lines; $I_x \rightarrow 2I_xS_y$ – blue dashed lines; $I_x \rightarrow 2I_xS_x$ – red dashed lines. Offset values in kHz: 0; 0; CSA values in kHz: 0, 0.

On the basis of Fig. 6b and d, we conclude that transfer of the signal from $I_x$ to $I_x$ is more complicated than from $I_x$ to $S_x$, although the same results are obtained in the end of two rotor periods. To define via which operators the homonuclear signal is transferred from one spin to another, we consider the amplitude of some operator that is generated as a result of another operator and evolution through pulses or dealsys, $t(\pi, x)$, $del_1, t(\pi, y)$, and $del_2$. For simplicity, we first consider the $IS$ spin system. Table 1 consists of four subsections, divided with different colors. The first (black color), second (green color), third (blue color) and fourth (red color) subsections represent the amplitudes of the operators, $I_x, S_x$,

$2I_xS_y, 2I_xS_x$, measured at four points when the initial operators are $I_x, S_x, 2I_xS_y, 2I_xS_x$, respectively. The used simulated parameters were as in Fig. 7a.

| Op  | $\pi_x$ | $del_1$ | $\pi_y$ | $del_2$ | $\pi_x$ | $del_1$ | $\pi_y$ | $del_2$ | $\pi_x$ | $del_1$ | $\pi_y$ | $del_2$ | $\pi_x$ | $del_1$ | $\pi_y$ | $del_2$ |
|-----|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| $I_x$ | 0.95   | 1       | 0.95    | 1       | 0.31    | 0       | 0.31    | 0       | 0       | 0       | 0.31    | 0       | 0       | 0       | 0       |
| $S_x$ | 0       | 0       | 0       | 0       | 0.31    | 0       | 0.31    | 0       | -       | 0.95    | 0       | 0       | 0.31    | 0       | 0       | 0       |
| $2I_xS_y$ | -   | 0.31  | 0       | 0       | 0.31    | 0       | 0.31    | 0       | -       | 0.95    | 0       | 0       | 0.31    | 0       | 0       | 0       |
| $2I_xS_x$ | 0     | 0     | 0       | 0.31    | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0.95    | 0       |

For example, the path $\pi_x \rightarrow I_x \rightarrow \pi_y \rightarrow I_x \rightarrow I_x$ gives the amplitude of $-0.95 \cdot 1 \cdot (-0.95) \cdot 1 = 0.9$ (the bold font in the Table 1), which equals to the amplitude of the $I_x$ operator at the end of $2T_R$ in Fig. 6a (black line). The path $\pi_x \rightarrow I_x \rightarrow I_x \rightarrow I_x \rightarrow I_x \rightarrow I_x$ gives the amplitude of $-0.95 \cdot 1 \cdot (-0.95) \cdot 0 = 0$. 19
If we analyze all possible 64 paths, we will find only one path, connecting \( I_z \) and \( S_z \) operators: 
\[
\pi_{x} \rightarrow 2I_xS_y \xrightarrow{\text{del}_1} 2I_xS_y \xrightarrow{\pi_{y}} S_z \xrightarrow{\text{del}_2} S_z
\]
with nonzero amplitude of 
\[
-0.31 \cdot 1 \cdot (-0.31) \cdot 1 = 0.097.
\]

In the same way we create the table for the \( I_z \) spin system (Table 2).

**Table 2** The microscopic amplitudes (Euler angles: 184°; 141°; 349°) of the operators (marked with bold font, the first column) in the end of four time points: \( \pi_{x} \) – in the end of first pulse; \( \text{del}_1 \) – in the end of first delay; \( \pi_{y} \) – in the end of second pulse; \( \text{del}_2 \) – in the end of second delay. The black, green, blue and red subsections represent the amplitudes with the initial operators \( I_{41}, I_{52}, 2I_{41}I_{52}, 2I_{51}I_{52} \), respectively. The simulated parameters were as in Fig. 7a.

<table>
<thead>
<tr>
<th>Op</th>
<th>( I_{41} )</th>
<th>( I_{52} )</th>
<th>( 2I_{41}I_{52} )</th>
<th>( 2I_{51}I_{52} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \pi_{x} )</td>
<td>-0.96</td>
<td>-0.96</td>
<td>-0.21</td>
<td>-0.21</td>
</tr>
<tr>
<td>( \text{del}_1 )</td>
<td>0.96</td>
<td>0.96</td>
<td>0.96</td>
<td>0.96</td>
</tr>
<tr>
<td>( \pi_{y} )</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>( \text{del}_2 )</td>
<td>0.21</td>
<td>0.21</td>
<td>0.21</td>
<td>0.21</td>
</tr>
</tbody>
</table>

Unlike IS spin system, there are 64 paths with nonzero amplitudes via which the signal is transferred from operator \( I_{41} \) to operator \( I_{52} \) during the first two rotor periods. 64 paths can be divided into four groups.

The first group contains eight paths with combinations of \( I_{41}, I_{52} \) operators only. For example, the path \( I_{41} \rightarrow I_{52} \xrightarrow{\text{del}_1} I_{52} \xrightarrow{\pi_{y}} I_{52} \xrightarrow{\text{del}_2} I_{52} \) has 0.0393 amplitude, whereas the path \( I_{51} \)

\[
\pi_{x} \rightarrow I_{52} \xrightarrow{\text{del}_1} I_{52} \xrightarrow{\pi_{y}} I_{52} \xrightarrow{\text{del}_2} I_{52} \]
has -0.0195 amplitude. The total amplitude of this group is 0.03920388.

The second group contains 24 paths where each of the paths contains one of the operators \( 2I_{41}I_{52} \) or \( 2I_{51}I_{52} \). For example, the path 
\[
I_{41} \rightarrow I_{52} \xrightarrow{\pi_{y}} I_{52} \xrightarrow{\text{del}_1} I_{52} \xrightarrow{\pi_{y}} 2I_{41}I_{52} \xrightarrow{\text{del}_2} I_{52}
\]
has -0.0393 amplitude, whereas the path 
\[
I_{41} \rightarrow I_{52} \xrightarrow{\pi_{y}} I_{52} \xrightarrow{\text{del}_1} I_{52} \xrightarrow{\pi_{y}} 2I_{51}I_{52} \xrightarrow{\text{del}_2} I_{52}
\]
has 0.0195 amplitude. The total amplitude of this group is 0.0574702.
The third group contains 24 paths where each of the paths contains two of the operators \( 2I_xI_y \) or \( 2I_yI_x \). For example, the path \( I_{21} \xrightarrow{\pi_x} 2I_yI_x \xrightarrow{det_z} 2I_yI_x \xrightarrow{\pi_y} I_x \xrightarrow{det_z} I_{22} \) has 0.0393 amplitude, whereas the path \( I_{21} \xrightarrow{\pi_x} 2I_yI_x \xrightarrow{det_z} I_x \xrightarrow{\pi_y} 2I_yI_x \xrightarrow{det_z} I_{22} \) has -0.000179 amplitude. The total amplitude of this group is 0.13445.

The fourth group contains eight paths where each of the paths contains three instances of the operators \( 2I_xI_y \), \( 2I_yI_x \). For example, the path \( I_{21} \xrightarrow{\pi_x} 2I_yI_x \xrightarrow{det_z} 2I_yI_x \xrightarrow{\pi_y} 2I_yI_x \xrightarrow{det_z} I_{22} \) has 0.0393 amplitude, whereas the path \( I_{21} \xrightarrow{\pi_x} 2I_yI_x \xrightarrow{det_z} 2I_yI_x \xrightarrow{\pi_y} 2I_yI_x \xrightarrow{det_z} I_{22} \) has 0.0197 amplitude. The total amplitude of this group is -0.0191567.

The total amplitude of all four groups at the time point \( 2T_R \) is 0.097, which is the same as for \( IS \) spin system.

Considering the paths of the \( I_2 \) spin system during \( 2T_R \) of time, we found a number of paths where the signal was transferred directly from \( I_{x1} \) to \( I_{x2} \) operators and not via \( I_{x1}I_{y2} \) and \( I_{x2}I_{y1} \) operators. However, it showed the microscopic amplitude at one time point only. We can calculate the powder amplitude of these paths, \( A_{22} \), as a function of the mixing time and compare with the total transferred signal from \( I_{x1} \) to \( I_{x2} \). For simplicity of the calculations, we take into account the paths where the jump from the operator \( I_{x1} \) to \( I_{x2} \) occurs only once:

\[
I_{21} \xrightarrow{\pi_x} \ldots I_{x1} \xrightarrow{t_k} I_{x2} \xrightarrow{\pi_{x+k+1}} \ldots I_{x2}.
\]

or the paths which contain three jumps between these two operators:

\[
I_{21} \xrightarrow{\pi_x} \ldots I_{x1} \xrightarrow{t_k} I_{x2} \xrightarrow{t_{x+k+1}} \ldots I_{x2} \xrightarrow{\pi_{x+k+1}} I_{x1} \xrightarrow{\pi_{x+k+m+1}} \ldots I_{x2} \xrightarrow{\pi_{x+k+m}} \ldots I_{x2}.
\]

In Fig. 9 we compare the total transferred signal (solid lines) with the direct transferred signal (dashed lines).
Under on resonance condition (Fig. 9, blue lines), the contribution of the direct transferred signal to the total is very small ($A_{zz}$, blue dashed line) and reaches ~2% of the starting signal. Addition of offset values (red lines) increases the contribution of this transfer (red dashed line) to the total signal (red solid line), where it reaches ~16% of the starting signal. However, as can be seen the major transfer of the signal from $I_{z1}$ to $I_{z2}$ occurs via $I_{x1}I_{y2}$ and $I_{y1}I_{x2}$ operators for both cases.

Fig. 9 The simulated total (solid lines) and the direct (dashed lines) transferred signals from operator $I_{z1}$ to $I_{z2}$ as a function of the mixing time. $\nu_{D,S} = 15$ kHz, 10 kHz of MAS and 65 kHz of rf-field. Blue lines: Offset values in kHz: 0, 0; CSA values in kHz: 0, 0. Red lines: Offset values in kHz: 2, -7; CSA values in kHz: 9.2, 2.5. The XY8 phase cycling was used.

Experiments

Fig. 10 shows a 1D heteronuclear fp-RFDR sequence. The sequence consists of two $\pi/2$-pulses on the $^1$H channel, fp-RFDR block, $\pi/2$-pulse on the $^{13}$C/$^{15}$N channel and detection with proton decoupling.
Fig. 10 1D fp-RFDR pulse sequence. The sequence consists of two π/2-pulses on the 1H channel, fp-RFDR block (a train of π-pulses every one rotor period on both channels), π/2-pulse on the 13C/15N channel and detection with proton decoupling. The phases of the π/2-pulses are \( \phi_1 = x; \phi_2 = -x,x; \phi_3 = x, x,-x,-x; \phi_{acq} = x, -x, -x, x \). π-pulses on both channels follow the XY8 scheme (Terry Gullion, Baker, and Conradi 1990). During acquisition, SW-TPPM decoupling is applied on the proton channel to narrow the detected resonances (Thakur, Kurur, and Madhu 2006).

Figs. 11, 12, 13 show 1D \( ^1 \text{H-}^{[13] \text{C}} \) fpRFDR spectra recorded with different values of the mixing time (thick blue lines), when the carbon reference frequency was set to 172 ppm, 120 ppm and 40 ppm, respectively. Red lines represent \( ^1 \text{H-}^{[13] \text{C}} \) cross polarization (CP) spectrum at 0.8 ms of mixing time.

When the carbon reference frequency is set to 172 ppm (Fig. 11), the proton magnetization is mostly transferred to the carbonyl/carboxyl and alkene/aromatic groups. Under such conditions, the carbonyl/carboxyl signal is increased with increasing mixing time, whereas the fp-RFDR signals of the other groups are decreased. The cyan spectrum in Fig. 12c represents the control experiment – π-pulses were not applied on the \( ^1 \text{H} \) channel during fp-RFDR and therefore zero transferred signal was measured.
Fig. 11 1D 1H-{13C} ramped CP (rCP) spectrum (red line, 0.8 ms of the mixing time) and fp-RFDR spectra under different values of the mixing time: 320 us (a), 480 us (b) and 640 us (c). The cyan spectrum in (c) represents fp-RFDR experiment, when π-pulses were not applied on the 1H channel. The Carbon reference frequency was set up on 172 ppm. 100 kHz MAS was used. The experimental parameters are shown in Table 3.

When the carbon reference frequency is set to 120 ppm (Fig. 12), the proton magnetization is also mostly transferred to the carbonyl/carboxyl and alkene/arylromatic groups. At 320 us of transfer time, we observe an asymmetry in the excitation of the alkene/arylromatic carbons – the fpRFDR peaks between 120 and 110 ppm have smaller intensities compared to CP excitation, whereas the fp-RFDR peaks between 135 and 125 ppm have larger intensities (Fig. 12b, inset). In general, the transfer is competitive with CP for the aromatic region.
Fig. 12 $^{13}$C fpRFDR spectra recorded at different mixing times of 160 us (a), 320 us (b), 480 us (c) and 640 us (d) compared with 0.8 ms CP. The carbon reference frequency was set to 120 ppm. The expansion in (b) shows aromatic signals between 150 and 100 ppm. 100 kHz of MAS was used. The experimental parameters are shown in Table 3.

When the carbon reference frequency is set on the 40 ppm (Fig. 13), the proton magnetization is mostly transferred to the aliphatic groups. Short mixing times result in optimal transfer.

Fig. 14 shows 1D $^1$H-$^{15}$N fp-RFDR spectra at different mixing times (thick blue lines), when the nitrogen reference frequency was set to 118 ppm. Red lines show a 1 ms $^1$H-$^{15}$N CP spectrum. Optimal transfer occurs at around 400 microseconds.
The first RFDR experiment via a longitudinal exchange was demonstrated in 1992 by Bennet et all (A. E. Bennett et al. 1992). It has since become one of the routine MAS solid-state NMR mixing sequences. Using average Hamiltonian theory, the theoretical and the simulated descriptions of this sequence were demonstrated in many articles (A. E. Bennett et al. 1992; Ishii 2001; Nishiyama, Zhang, and Ramamoorthy 2014; R. Zhang et al. 2015; Brinkmann, Schmedt auf der Günne, and Levitt 2002; Ji et al. 2020). In those theoretical descriptions the width of the duty factor was considered as a main source of the fp-RFDR transferred signal, whereas a role of the phase cycling was discussed in the context of a reduction in deleterious effects of resonance offsets and pulse errors (A. E. Bennett et al. 1992; Ishii 2001). Through AHT, the evolution of the spin system under RFDR has been previously calculated at specific time points.

In this article we showed a numerical investigation of the fpRFDR sequence. Using a three spin system, we showed that depending on the conditions the total dipolar Hamiltonian could be replaced by two different simplified model Hamiltonians, which described the same evolution of the spin system at specific time points as a total dipolar Hamiltonian. For the first case, small differences between offset values compared with the MAS rate, a good model Hamiltonian was the secular part only, \(I_xI_z\) (Eq. 2.2a). For the second case of larger offsets, the flip-flop Hamiltonian (Eq. 2.2c) could be considered as a model Hamiltonian. The conclusion for the first case indicated the possibility for a heteronuclear fp-RFDR transfer. Heteronuclear fp-RFDR was demonstrated experimentally for both proton-carbon and proton-nitrogen transfer. While the transfer efficiency was not as high as for ramped CP, a comparable transfer efficiency was observed for aromatic signals. Since heteronuclear RFDR simultaneously recouples homonuclear dipolar interactions, it may still be useful where longer relayed transfers are desired.

Using two model spin systems we investigated the macroscopic amplitudes of the heteronuclear and homonuclear operators and showed that for small offset differences the evolution of the homonuclear and heteronuclear spin systems could be described with the same set of the operators with the same amplitudes, if one looks only at the end of two rotor periods. However, the evolutions of the homonuclear
and heteronuclear operators were completely different, when their amplitudes were simulated within the 2 rotor period block.

We demonstrated with simulations and provided the theoretical proof that XY phase cycling of $\pi$-pulses has a crucial role in the transfer of the homonuclear and heteronuclear fp-RFDR signals. With phase cycling of XX or $X\bar{X}$ the fpRFDR transfer does not occur, except for cases when the offset differences are comparable with the MAS rate.

We considered the paths and the operators, which were involved in transfer of the signal during the first two rotor periods. For the heteronuclear spin system we found only one path with nonzero amplitude, whereas for the homonuclear spin system the signal was transferred via 64 paths with nonzero amplitudes. However, by the end of two rotor periods the amplitudes of the homonuclear operators coincided with the amplitudes of the heteronuclear operators.

**Experimental methods**

**Sample preparation:** 100% back bone protonated Transamination ($\alpha$-PET SH3) was prepared by Movellan with the protocol described in Ref. [(Movellan et al. 2019)].

**Simulations:** RFDR simulations were performed with in-house MATLAB scripts using numerical solution of the experiment. The description of the simulations can be found in the ‘Theory’ section of this article.

**Solid state NMR spectroscopy:** ramped CP and fp-RFDR $H^{13}C$ / $H^{15}N$ experiments were performed at a 22.3 T (950 MHz) Bruker Avance III spectrometer using a Bruker 0.7 mm $^1H$-$^{13}C$-$^2D$-$^{15}N$ probe. In all experiments 100 kHz of MAS was used and the temperature was set to 260 K. 18.5 kHz SW-TPPM (Thakur, Kurur, and Madhu 2006) with 25 us pulses was used during the acquisition. Tables 3 and 4 summarize the applied experimental parameters.

**Table 3 Summary** of the experimental parameters used in the rCP (the start and the end values are shown) and fpRFDR $H^{13}C$ experiments.
Table 4 Summary of the experimental parameters used in the rCP (the start and the end values are shown) and fpRFDR H\(^{15}\)N experiments.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>rCP</th>
<th>fpRFDR</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^1)H (kHz)</td>
<td>118-139</td>
<td>50.2</td>
</tr>
<tr>
<td>(^{13})C (kHz)</td>
<td>30.2</td>
<td>49.31</td>
</tr>
<tr>
<td>transfer time</td>
<td>1 ms</td>
<td>240, 400, 560 μs</td>
</tr>
<tr>
<td>NS</td>
<td>2000</td>
<td>2000</td>
</tr>
<tr>
<td>D1 (s)</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>AQ (s)</td>
<td>0.0135168</td>
<td>0.0135168</td>
</tr>
<tr>
<td>SW (Hz)</td>
<td>75757.58</td>
<td>75757.58</td>
</tr>
</tbody>
</table>

NS – Number Scans; D1 – a recycle delay; AQ – the acquisition time; SW – the spectral width.
Fig. A1 shows the fpRFDR signals of the $I_3$ (Fig. A1a and c) and $IS_2$ (Fig. A1b and d) spin systems obtaining with MATLAB simulations (solid lines) and SIMPSON simulations (grey dashed lines).

MATLAB and SIMPSON simulated curves provide the same behavior of the starting and transferred signals under different experimental conditions: 10 kHz (Fig. A1a and b) and 90 kHz (Fig. A1 c and d) of MAS rates. $IS_2$ spin system contains the heteronuclear and homonuclear dipolar interactions, whereas $I_3$ spin system contains homonuclear dipolar interactions only. Under slow MAS rate of 10 kHz we detect low transferred signals between weakly bounded spins – $I_{z2}$ and $I_{z1}$ (black line) and $I_{z3}$ and $I_{z1}$ (purple line) for $I_3$ spin system (Fig. A1a) and much weaker signals for $IS_2$ spin system - $S_{z2}$ and $I_{z1}$ (black line) and $S_{z3}$ and $I_{z1}$ (purple line), Fig. A1b. However, under 90 kHz of MAS rate, the simulations provides the same results for $I_3$ (Fig. A1c) and $IS_2$ (Fig. A1d) spin systems.

Fig. A1 Simulated $I_3$ (a, c) and $IS_2$ (b, d) spin systems under 10 kHz (a, b) and 90 kHz (c, d) of MAS with 65 kHz of rf-field, dipolar values of $\nu_{12} = 66$ Hz, $\nu_{13} = 150$ Hz, $\nu_{23} = 2.15$ kHz, the offset values of $\Omega_1 = -8$ kHz, $\Omega_2 = 9$ kHz, $\Omega_3 = -7$ kHz.
and CSA values of \( \nu_{\text{CSA},1} = 9.2 \text{ Hz}, f_{\nu_{\text{CSA},2}} = 2.5 \text{ Hz}, f_{\nu_{\text{CSA},3}} = 8 \text{ kHz} \). The solid lines represent fp-RFDR starting and transferred signals obtaining with MATLAB simulations, whereas the dashed signals were obtained with SIMPSON simulations.

(a), (c): \( I_z^2 \rightarrow I_z^2 \) (red lines); \( I_z^2 \rightarrow I_z^3 \) (blue lines); \( I_z^2 \rightarrow I_z^1 \) (black lines); \( I_z^3 \rightarrow I_z^1 \) (purple lines). (b), (d): \( S_z^2 \rightarrow S_z^2 \) (red lines); \( S_z^2 \rightarrow S_z^3 \) (blue lines); \( S_z^2 \rightarrow I_z^1 \) (black lines); \( S_z^3 \rightarrow I_z^1 \) (purple lines). The values of the dipolar interactions and 10 kHz of MAS was taken from Ref. [Bayro et al. 2009]. XY8 phase cycling was used.

Fig A2 compares the starting and transferred signals obtained with the full dipolar Hamiltonian (Eq. 2.1) or simulating with the secular dipolar Hamiltonian (Eq. 2.2), when CSA values (Fig. A2a) or offset values (Fig. A2b) are added to the simulations. The CSA values themselves (Fig. A2a) have very small influence on the evolution of the spin system and therefore the secular dipolar Hamiltonian (red lines) provides the same evolution of the spin system as the full dipolar Hamiltonian (black lines). The main influence comes from offset values as shown in Fig. A2b. Under these conditions, the red and black lines do not coincide.

Fig. A2 Simulated \( I_z \) spin system under 10 kHz of MAS and 65 kHz of rf-field, dipolar coupling values of \( \nu_{12} = 66 \text{ Hz}, \nu_{13} = 150 \text{ Hz}, \nu_{23} = 2.15 \text{ kHz} \). Axis Y shows the intensities of the starting and transferred signals between different operators: \( I_z^2 \rightarrow I_z^2 \) (the dotted lines); \( I_z^2 \rightarrow I_z^3 \) (the dashed lines); \( I_z^2 \rightarrow I_z^1 \) (the solid lines); \( I_z^3 \rightarrow I_z^1 \) (the dashed-dotted lines). The black lines represent the signals, simulating with the full dipolar Hamiltonian (Eq. 2.1) with zero values of offset and CSA. The red lines represent the signals, simulating with the secular model Hamiltonian (Eq. 2.2a). (a) Offset values: 0; CSA values: \( \nu_{\text{CSA},1} = \)

9.2 Hz, \( f_{\nu_{\text{CSA},2}} = 2.5 \text{ Hz}, f_{\nu_{\text{CSA},3}} = 8 \text{ kHz} \). (b) \( \Omega_1 = -8 \text{ kHz}, \Omega_2 = 9 \text{ kHz}, \Omega_3 = -7 \text{ kHz} \); CSA values: 0.
Appendix B

For the theoretical proof of zero fpRFDR signal at specific time points when XX phase cycling is used, we can consider the transfer of the signal from spin \( I_1 \) to spin \( I_2 \) at the end of one rotor period. The measured operator at this time is described with the Eqn.:

\[
\langle I_z_2 \rangle (T_R) = Tr\{I_z_2 U(T_R) I_{x_1} U^{-1}(T_R)\}. \tag{B1}
\]

For simplicity, we take into account the dipolar interaction + rf-field during \( \pi \)-pulse and the dipolar interaction only during the delay. In that case the unitary operator, \( U(T_R) \) is written as follow:

\[
U(T_R) = U_2 U_1; \quad U_2 = \hat{T} \exp\left\{ \int_{t_p}^{T_R} dt \omega_{D,12}(t)(3I_{x_1}I_{z_2} - I_{z_1}I_2) \right\} \tag{B2}
\]

\[
U_1 = \hat{T} \exp\left\{ \int_0^{t_p} dt \left[ \omega_{D,12}(t)(3I_{x_1}I_{z_2} - I_{z_1}I_2) + \omega_{rf}(I_{x_1} + I_{x_2}) \right] \right\}
\]

where \( \hat{T} \) is a Dyson operator and \( \omega_{D,12}(t) \) is a periodic dipolar time dependent function (Olejniczak, Vega, and Griffin 1984) between spins \( I_1 \) and \( I_2 \). Firstly, we can simplify Eq. B2 omitting the scalar product, \( I_1 I_2 \), since it communicates with other parts of the Hamiltonian:

\[
[I_1 I_2, I_{x_1}I_{z_2}] = [I_1 I_2, I_{x_1} + I_{x_2}] = 0. \tag{B3}
\]

and the dipolar function is periodic - \( \int_0^{T_R} dt \omega_{D,12}(t)I_1 I_2 = 0 \). Eq. B2 can be written as follow:

\[
U(T_R) = U_2 U_1; \quad U_2 = \hat{T} \exp\left\{ \int_{t_p}^{T_R} dt \omega_{D,12}(t)3I_{x_1}I_{z_2} \right\} \tag{B4}
\]

\[
U_1 = \hat{T} \exp\left\{ \int_0^{t_p} dt \left[ \omega_{D,12}(t)3I_{x_1}I_{z_2} + \omega_{rf}(I_{x_1} + I_{x_2}) \right] \right\}
\]

The next step is the rotation of the all operators by \( 90^\circ \) around axis \(-y\):

\[
I_{x_1}, I_{x_2}, I_{x_1}I_{z_2}, (I_{x_1} + I_{x_2}) \xrightarrow{90^\circ-y} -I_{x_1}, -I_{x_2}, I_{x_1}I_{z_2}, (I_{x_1} + I_{x_2}). \tag{B5}
\]

Substituting Eq. B5 into Eqs. B1 and B4, the modified Eq. B1 is:

\[
\langle I_{z_2} \rangle (T_R) = Tr\{I_{z_2}U(T_R) I_{x_1} U^{-1}(T_R)\}. \tag{B6}
\]

whereas the modified Eq. B4 is:
\( U(T_R) = U_2 U_1: \)

\[
U_2 = \hat{T} \exp \left\{ \int_{t_p}^{t_R} dt \omega_D,12(t) 3I_x \right\}
\]

\[
U_1 = \hat{T} \exp \left\{ \int_{t_p}^{t_R} dt \left[ \omega_D,12(t) 3I_x + \omega_{rf} (I_{z1} + I_{z2}) \right] \right\}
\]

The operators in Eqs. B6 and B7 can be rewritten with fictitious spin \( \frac{1}{2} \) operator formalism(Vega 1978):

\( 2I_x I_x = I_x^{(1.4)} + I_x^{(2.3)}, \)

\( (I_{z1} + I_{z2}) = 2I_x^{(1.4)} . \)

Therefore, Eqs. B6 and B7 can be written as follow:

\[
\langle I_{z2}(T_R) \rangle = Tr \{ I_{z2} U(T_R) I_{z1} U^{-1}(T_R) \}, \quad (B9)
\]

\[
U(T_R) = U_2 U_1: \quad \hat{T} \left\{ \int_{t_p}^{t_R} dt \omega_D,12(t) 3 \right\}
\]

\[
U_2 = \hat{T} \left\{ \int_{t_p}^{t_R} dt \left[ \omega_D,12(t) 3I_x + \omega_{rf} (I_{z1} + I_{z2}) \right] \right\}.
\]

Since the operator \( I_x^{(2.3)} \) communicates with other operators and the dipolar function is periodic –

\[
\int_0^{T_R} dt \omega_D,12(t) I_x^{(2.3)} = 0 \quad \text{– the Eqs. B9 and B10 can be rewritten as:}
\]

\[
\langle I_{z2}(T_R) \rangle = Tr \left\{ I_{z2} U^{(1.4)}(T_R) I_{z1} (U^{(1.4)})^{-1}(T_R) \right\}.
\]

\[
U^{(1.4)}(T_R) = U_2^{(1.4)} U_1^{(1.4)}: \quad \hat{T} \left\{ \int_{t_p}^{t_R} dt \omega_D,12(t) 3I_x^{(1.4)} \right\}
\]

\[
U_2^{(1.4)} = \hat{T} \left\{ \int_{t_p}^{t_R} dt \omega_D,12(t) 3I_x^{(1.4)} + \omega_{rf} 2I_x^{(1.4)} \right\}.
\]

On the basis of the fictitious spin \( \frac{1}{2} \) operator formalism(Vega 1978), the next properties are always performed:

\[
2I_x I_x^{(1.4)} 2I_x j = I_x^{(2.3)}, \quad (B11)
\]

\[
2I_x I_j^{(1.4)} 2I_x j = -I_j^{(2.3)}, \quad j = 1,2.
\]

On the basis of these properties Eqs. B9 and B10 are:

\[
\langle I_{z2}(T_R) \rangle = Tr \left\{ I_{z2} I_{z1} U^{(2.3)}(T_R) (U^{(1.4)})^{-1}(T_R) \right\}.
\]

\[
U^{(2.3)}(T_R) = U_2^{(2.3)} U_1^{(2.3)}: \quad \hat{T} \left\{ \int_{t_p}^{t_R} dt \omega_D,12(t) 3I_x^{(2.3)} \right\}
\]

\[
U_2^{(2.3)} = \hat{T} \left\{ \int_{t_p}^{t_R} dt \omega_D,12(t) 3I_x^{(2.3)} - \omega_{rf} 2I_x^{(2.3)} \right\}.
\]

On the basis of Eq. B8 the product of \( I_{z2} I_{z1} \) can be rewritten and therefore Eq. B12 is:
\[ \langle I_{z2} \rangle (T_R) = 0.5 T_R \left\{ \left( I_x^{(1,4)} + I_x^{(2,3)} \right) U^{(2,3)} (T_R) \left( U^{(1,4)} \right)^{-1} (T_R) \right\} = \]
\[ = 0.5 T_R \left\{ I_x^{(2,3)} U^{(2,3)} (T_R) \right\} + 0.5 T_R \left\{ I_x^{(1,4)} (U^{(1,4)})^{-1} (T_R) \right\} . \]

The next step will be usage of the mentioned properties of fictitious spin \( \frac{1}{2} \) operator formalism (Eq. B11):
\[ \langle I_{z2} \rangle (T_R) = 0.5 T_R \left\{ I_x^{(2,3)} U^{(2,3)} (T_R) \right\} + 0.5 T_R \left\{ I_x^{(2,3)} (U^{(2,3)})^{-1} (T_R) \right\} . \] (B15)

The last step will be the usage the next property:
\[ -2I_y^{(2,3)} I_x^{(2,3)} = I_x^{(2,3)}, \]
\[ -2I_y^{(2,3)} I_x^{(2,3)} = I_x^{(2,3)}. \] (B16)

Substituting Eq. B11 into Eq. B13, then the modified Eq. B13 into Eq. B15 and taking into account that \( 2I_y^{(2,3)} I_y^{(2,3)} = 1^{(2,3)} \), the transferred signal is:
\[ \langle I_{z2} \rangle (T_R) = -0.5 T_R \left\{ I_x^{(2,3)} (U^{(2,3)})^{-1} (T_R) \right\} + 0.5 T_R \left\{ I_x^{(2,3)} (U^{(2,3)})^{-1} (T_R) \right\} = 0. \] (B17)

Since the transferred signal is zero at mixing the time of one rotor period, it is always zero at integer multiples of rotor periods.

---

588 https://doi.org/10.1063/1.3089370.


Nishiyama, Yusuke, Michal Malon, Yuji Ishii, and Ayyalusamy Ramamoorthy. 2014. “3D 15N/15N/1H Chemical Shift Correlation Experiment Utilizing an RFDR-Based 1H/1H Mixing Period at 100kHz MAS.” Journal of Magnetic Resonance 244 (July): 1–5. https://doi.org/10.1016/j.jmr.2014.04.008.


