Comments on Jean's reciprocity. Tom Barbara 5/13/21

Here are further aspects of my previous comments on the Jeans reciprocity that this submission uses in their design efforts. For each *prescribed* source $\mathbf{M}_1$ and $\mathbf{M}_2$ we have (in Gaussian units)

$$\boldsymbol{B}_i - \mathbf{H}_{\boldsymbol{i}} = 4\pi\, \mathbf{M}_{\boldsymbol{i}} \qquad (1)$$

$$\boldsymbol{\nabla} \times \boldsymbol{H}_{\boldsymbol{i}} = 0$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{B}_{\boldsymbol{i}} = 0$$

It is important to emphasize that the $\mathbf{M}_i$ are prescribed sources, that is that the magnetizations are saturated or fixed, as discussed in Jackson's book on E&M.

We can take scalar products of (1) for each source and $\mathbf{H}_j$ for the other, and take their difference

$$\mathbf{H}_{\mathbf{2}} \cdot (\boldsymbol{B}_1 - \mathbf{H}_1) = 4\pi\, \mathbf{H}_{\mathbf{2}} \cdot \mathbf{M}_1$$

$$\mathbf{H}_{\mathbf{1}} \cdot (\boldsymbol{B}_2 - \mathbf{H}_{\mathbf{2}}) = 4\pi\, \mathbf{H}_{\mathbf{1}} \cdot \mathbf{M}_{\mathbf{2}}$$

The terms in **H** for each source are thereby eliminated. After integrating over all space, the mixed field terms vanish by the fact that they are "Helmholtz pairs", so that we are left with

$$\int \boldsymbol{d^3x}\ \mathbf{H}_{\mathbf{1}} \cdot \mathbf{M}_2 \ = \ \int \boldsymbol{d^3x}\ \mathbf{H}_{\mathbf{2}} \cdot \mathbf{M}_1$$

I believe this is a clearer statement of the Jeans reciprocity. Of course the validity of this theorem does require linear superposition which follows from the assumption that the sources are prescribed.