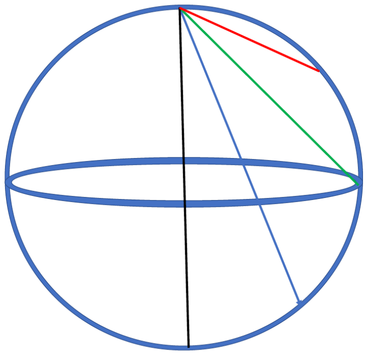
Trajectories of classical magnetization vectors under the effects of relaxation

The point I have tried to make, in response to Malcolm's brilliant paper with Christian Bengs (mr 2021-26) on Lindblad vs. Liouville equations, seems to have been lost in a "furious" debate. It must be feared that even Malcolm has become a victim of the current fashion of looking at pictures rather than reading a simple text, *a fortiori* trying to understand an equation. Let me illustrate my idea graphically, for whatever it may be worth:



All four trajectories start on the intersection of the zOy plane with the surface of Bloch's famous sphere, the red one at the latitude of Paris, the green one on the equator, the blue one further South, while the black line starts on the South pole. It is hardly surprising for practitioners of NMR that all trajectories end up at the North pole, regardless of their initial position. The remarkable feature is that, provided the so-called extreme narrowing condition $T_1 = T_2$ is fulfilled, all trajectories follow the *shortest path* along a straight line. Writing as usual $\Delta M_z(t) = M_z(t) - M_z(t = \infty)$ to denote the deviation of the longitudinal component from its thermal equilibrium, and the modulus of the transverse components $M_{yx} = [M_x^2 + M_y^2]^{1/2}$, one has the same exponential 'equations of motion' for both components:

$$M_{yx}(t) \;=\; M_{xy}(\mathrm{t{=}0})\;\; \exp\{-t/T_2\}$$
$$\Delta M_z(\mathrm{t}) \;=\; \Delta M_z(\mathrm{t{=}0})\;\; \exp\{-t/T_2\}$$

Normalizing with respect to their initial values one obtains

$$M_{yx}(t)/M_{xy}(\mathrm{t{=}0}) \;=\; \exp\{-t/T_2\}$$
$$\Delta M_z(\mathrm{t})/\Delta M_z(\mathrm{t{=}0}) \;=\; \exp\{-t/T_2\}$$

It is clear that these ratios are equal for all times t. This leads to a linear relationship:

$$\Delta M_z(t) = b\, M_y(\mathrm{t}) \qquad \text{with the slope } b = [\Delta M_z(t{=}0)/M_{xy}(t{=}0)]$$

I have two questions about Malcolm's and Christian's preprint (mr 2021-26): (i) is their trajectory in Fig. 8 exactly linear as is seems to the naked eye, and (ii) if so, could there be a similar reason as for the Bloch equations, bearing in mind that their work is not concerned with any transverse components, but only with populations of eigenstates? Or is analogy between the two straight paths accidental?

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