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## Synthesis and characterisation of ligand C13 and corresponding Ln complexes

## 2-(Hydroxymethyl)-4-nitropyridine (1)



To a solution of commercially available 2-methyl-4-nitropyridine $N$-oxide ( $0.70 \mathrm{~g}, 4.5 \mathrm{mmol}$ ) in $\mathrm{CH}_{2} \mathrm{Cl}_{2}(14 \mathrm{~mL})$ was added trifluoracetic anhydride ( $1.2 \mathrm{~mL}, 9.0 \mathrm{mmol}$ ) in $\mathrm{CH}_{2} \mathrm{Cl}_{2}(2.1 \mathrm{~mL})$. The solution was stirred at room temperature for 72 hours, during which time the pale-yellow solution turned deep red in colour. The solvent was removed under reduced pressure and resulting orange oil was dissolved in $\mathrm{MeOH}(7.0 \mathrm{~mL})$ and saturated aq. $\mathrm{K}_{2} \mathrm{CO}_{3}$ solution was added until the pH was approximately 8 . The solution was stirred at room temperature for 24 hours. The solvent was removed under reduced pressure and the solid was partitioned between water $(30 \mathrm{~mL})$ and EtOAc $(30 \mathrm{~mL})$ and extracted with EtOAc $(2 \times 30 \mathrm{~mL})$. The organic layers were combined, dried over magnesium sulfate and concentrated under vacuum to give the crude material, which was purified by column chromatography (silica gel; $0-10 \% \mathrm{MeOH}$ in $\left.\mathrm{CH}_{2} \mathrm{Cl}_{2}\right)$ to give the desired methyl alcohol $1(0.225 \mathrm{~g}, 33 \%)$ as a pale-yellow solid. ${ }^{1} \mathrm{H}$ NMR ( $500 \mathrm{MHz}, \mathrm{CD}_{3} \mathrm{OD}$ ) $\delta: 8.77$ (d, $J=5.4 \mathrm{~Hz}, 1 \mathrm{H}$ ), 8.17 ( $\mathrm{s}, 1 \mathrm{H}$ ), 7.94 (dd, $J=5.4 \mathrm{~Hz}, 1 \mathrm{H}$ ), 4.78 (s, 2H), O-H signal not observed. ${ }^{13} \mathrm{C}$ NMR ( $125 \mathrm{MHz}, \mathrm{CD}_{3} \mathrm{OD}$ ) $\delta: 165.3,154.2,150.9,114.6$, 112.8, 63.9. HRMS (ESI + ) found $155.0406[\mathrm{M}+\mathrm{H}]^{+},\left[\mathrm{C}_{6} \mathrm{H}_{7} \mathrm{~N}_{2} \mathrm{O}_{3}\right]^{+}$requires 155.0451. The NMR data are in agreement with those reported previously (Gempf et al., 2013).

## (R)-2-Bromo- N -(1-phenylethyl)acetamide (2)



A solution of $(R)$-1-phenylethan-1-amine ( $6.4 \mathrm{~mL}, 50 \mathrm{mmol}$ ) in anhydrous $\mathrm{CH}_{2} \mathrm{Cl}_{2}(25 \mathrm{~mL})$ was added dropwise to a solution of bromoacetyl bromide ( $2.2 \mathrm{~mL}, 28 \mathrm{mmol}$ ) in anhydrous $\mathrm{CH}_{2} \mathrm{Cl}_{2}(75 \mathrm{~mL})$, at $0{ }^{\circ} \mathrm{C}$ under a nitrogen atmosphere. The reaction mixture was stirred for 2 hours and allowed to warm to room temperature. The reaction mixture was then washed with 2 M HCl solution ( $1 \times 60 \mathrm{~mL}$ ) followed by brine ( $2 \times 60 \mathrm{~mL}$ ). The organic layer was dried over magnesium sulfate and the solvent was removed under reduced pressure to give pure $(R)-2$ -bromo- $N$-(1-phenylethyl)acetamide $2(0.747 \mathrm{~g}, 40 \%)$ as a white solid. No further purification
was required. ${ }^{1} \mathrm{H}$ NMR ( $500 \mathrm{MHz}, \mathrm{CDCl}_{3}$ ) $\delta: 7.38-7.27(\mathrm{~m}, 5 \mathrm{H}), 6.77(\mathrm{~s}, 1 \mathrm{H}), 5.14-5.07(\mathrm{~m}$, $1 \mathrm{H}), 3.89-3.84(\mathrm{dd}, J=14.67 \mathrm{~Hz}, 6.18 \mathrm{~Hz}, 2 \mathrm{H}), 1.53(\mathrm{~d}, J=7.00 \mathrm{~Hz}, 3 \mathrm{H}) .{ }^{13} \mathrm{C}$ NMR ( 125 $\mathrm{MHz}, \mathrm{CDCl}_{3}$ ) $\delta: 164.7,142.4,128.9,127.7,126.2,49.7,29.3,21.7$. HRMS (ESI+): found $263.9995[\mathrm{M}+\mathrm{Na}]^{+},\left[\mathrm{C}_{10} \mathrm{H}_{12} \mathrm{NOBrNa}\right]^{+}$requires 264.0000.

## 2,2', $\mathbf{2}^{\prime \prime}$-(1,4,7,10-Tetraazacyclododecane-1,4,7-triyl)tris( $N$-(( $R$ )-1-phenylethyl)acetamide)

(3)


A solution of ( $R$ )-2-bromo- $N$-(1-phenylethyl)acetamide $2(0.703 \mathrm{~g}, 2.90 \mathrm{mmol}$ ) in anhydrous $\mathrm{CH}_{3} \mathrm{CN}(10 \mathrm{~mL})$ was added dropwise to a stirred mixture of cyclen $(0.200 \mathrm{~g}, 1.20 \mathrm{mmol})$ and potassium carbonate $(0.160 \mathrm{~g}, 1.20 \mathrm{mmol})$ in anhydrous $\mathrm{CH}_{3} \mathrm{CN}(50 \mathrm{~mL})$ under a nitrogen atmosphere. The mixture was heated to reflux for 4 hours then allowed to cool to room temperature overnight. Once complete consumption of $\mathbf{2}$ was observed by LCMS analysis, the reaction mixture was filtered and the solvent was removed under reduced pressure. The residue was taken up in water ( 300 mL ), adjusted to pH 3 and stirred for 30 minutes. EtOAc ( 150 mL ) was added and the biphasic mixture was transferred to a separatory funnel and vigorously shaken. The organic layer containing over-alkylated byproduct was removed. The aqueous layer was then washed with $\mathrm{CH}_{2} \mathrm{Cl}_{2}(2 \times 150 \mathrm{~mL})$ and the pH of the aqueous layer adjusted to 5.5 and washed with $\mathrm{CH}_{2} \mathrm{Cl}_{2}(2 \times 150 \mathrm{~mL})$. This process was repeated at pH 7 . The organic fractions containing desired product were combined, dried over magnesium sulfate and the solvent was removed under reduced pressure to give the desired product ( $190 \mathrm{mg}, 25 \%$ ) as a white solid. ${ }^{1} \mathrm{H}$ NMR ( $400 \mathrm{MHz}, \mathrm{CD}_{3} \mathrm{CN}$ ) $\delta: 7.87(\mathrm{~d}, J=7.8 \mathrm{~Hz}, 1 \mathrm{H}), 7.74(\mathrm{~d}, J=8.2 \mathrm{~Hz}, 2 \mathrm{H})$, 7.34-7.17 (m, 15H), 4.98-4.89 (m, 3H), 3.22-3.08 (m, 6H), 2.84-2.47 (m, 16H), 1.40-1.39 (m, 9H). ${ }^{13} \mathrm{C}$ NMR ( $100 \mathrm{~Hz}, \mathrm{CD}_{3} \mathrm{CN}$ ) $\delta: 171.6,171.5,145.6,145.4,129.5,129.4,128.0,127.9$, 127.3, 127.0, 60.1, 56.0, 55.1, 53.8, 51.4, 50.0, 49.8, 47.2, 22.8, 22.6. HRMS (ESI+): found $656.4283[\mathrm{M}+\mathrm{H}]^{+},\left[\mathrm{C}_{38} \mathrm{H}_{54} \mathrm{~N}_{7} \mathrm{O}_{3}\right]^{+}$requires 656.4288.

## Ligand C13



Step 1. To a solution of methyl alcohol $1(60 \mathrm{mg}, 0.38 \mathrm{mmol})$ and DIPEA ( $99 \mu \mathrm{~L}, 0.57 \mathrm{mmol}$ ) in anhydrous $\mathrm{CH}_{2} \mathrm{Cl}_{2}(4 \mathrm{~mL})$ was added methansulfonyl chloride ( $28 \mu \mathrm{~L}, 0.36 \mathrm{mmol}$ ) and the resulting mixture was stirred at room temperature for 2 hours. The reaction mixture was washed with brine ( 10 mL ) and the aqueous layer was extracted with $\mathrm{CH}_{2} \mathrm{Cl}_{2}(3 \times 10 \mathrm{~mL}$ ). The combined organic layers were dried over magnesium sulfate and concentrated under reduced pressure to give the mesylate ester ( $83 \mathrm{mg}, 94 \%$ ) as a yellow oil, which was used immediately in the next step.

Step 2. To a solution of macrocycle $\mathbf{3}(93 \mathrm{mg}, 0.14 \mathrm{mmol})$ in anhydrous $\mathrm{CH}_{3} \mathrm{CN}(4.5 \mathrm{~mL})$ was added potassium carbonate ( $42 \mathrm{mg}, 0.30 \mathrm{mmol}$ ) and the mixture was stirred for 5 minutes. The mesylate ester of $\mathbf{1}(40 \mathrm{mg}, 0.17 \mathrm{mmol})$ in anhydrous $\mathrm{CH}_{3} \mathrm{CN}(1.0 \mathrm{~mL})$ was added and the mixture was stirred at $60^{\circ} \mathrm{C}$ for 24 hours. The reaction mixture was then cooled to room temperature and centrifuged for 3 minutes. The solution was decanted, and the solid pellet were washed twice with $\mathrm{CH}_{3} \mathrm{CN}(10 \mathrm{~mL})$. The combined organic layers were concentrated under reduced pressure and the crude material was purified by column chromatography (silica gel; neat $\mathrm{CH}_{2} \mathrm{Cl}_{2}$ to $\left.9: 1 \mathrm{CH}_{2} \mathrm{Cl}_{2} / \mathrm{MeOH}\right)$ to give the desired ligand $\mathbf{C 1 3}(40 \mathrm{mg}, 35 \%)$ as a yellow solid. ${ }^{1} \mathrm{H}$ NMR ( $400 \mathrm{MHz}, \mathrm{CDCl}_{3}$ ) $\delta: 9.00-7.68(\mathrm{~m}, 3 \mathrm{H}) 8.13(\mathrm{~s}, 1 \mathrm{H}), 7.45-7.05(\mathrm{~m}, 17 \mathrm{H}), 4.87$ ( $\mathrm{t}, J=12.4,6.2 \mathrm{~Hz}, 1 \mathrm{H}), 4.72-4.59(\mathrm{~m}, 2 \mathrm{H}), 3.44-1.97(\mathrm{~m}, 2 \mathrm{H}), 1.44(\mathrm{t}, J=14.4,7.0 \mathrm{~Hz}, 6 \mathrm{H})$, $1.30(\mathrm{~d}, J=7.0 \mathrm{~Hz}, 3 \mathrm{H}){ }^{13} \mathrm{C}$ NMR ( $100 \mathrm{MHz}, \mathrm{CDCl}_{3}$ ) $\delta: 171.4,170.8,169.8,161.3,153.8$, $151.4,145.3,144.5,143.9,128.5,128.4,128.1,127.3,126.9,126.8,126.6,126.3,125.7,115.7$, $114.6,58.3,58.0,56.8,51.1,50.8,50.1,49.3,22.8,22.6,21.8$. HRMS (ESI+): found 814.4373 $[\mathrm{M}+\mathrm{Na}]^{+},\left[\mathrm{C}_{44} \mathrm{H}_{57} \mathrm{~N}_{9} \mathrm{O}_{5} \mathrm{Na}\right]^{+}$requires 814.4375 .

Ln.C13


To a solution of $\mathbf{C 1 3}(100 \mathrm{mg}, 0.13 \mathrm{mmol})$ in a mixture of $\mathrm{MeCN} / \mathrm{H}_{2} \mathrm{O}(1: 1,6 \mathrm{~mL})$ was added $\mathrm{LnCl}_{3} .6 \mathrm{H}_{2} \mathrm{O}$ ( 1.05 equiv., where $\mathrm{Ln}=\mathrm{Tm}, \mathrm{Y}$ ) and the reaction mixture was stirred at $70^{\circ} \mathrm{C}$ for 2 hours. Complete complexation was observed by LCMS analysis after this time. The organic solvent was removed under reduced pressure and the water was removed by freeze drying. The crude complexes, Tm.C13 and Y.C13, were obtained as white solids ( 120 mg , quant.) in each case. The complexes were purified by reverse-phase HPLC [XBridge C18 column, gradient: 0 $-50 \%$ methanol ( $0.05 \% \mathrm{v} / \mathrm{v}$ formic acid) in water ( $0.05 \% \mathrm{v} / \mathrm{v}$ formic acid), over 10 minutes at 17 mL per minute] to give Tm.C13 (20.5 mg, $16 \%$ ) and Y.C13 (20 mg, $17 \%$ ) as white solids. Analytical RP-HPLC analysis [XBridge C18 column, $100 \%$ water ( $0.1 \% \mathrm{v} / \mathrm{v}$ formic acid) for 5 minutes followed by a gradient of $0-50 \%$ methanol ( $0.1 \% \mathrm{v} / \mathrm{v}$ formic acid) over 10 minutes, at 0.7 mL per minute] revealed single peaks at $\mathrm{RT}=15.6$ minutes for $\mathbf{T m} . \mathbf{C 1 3}$, and RT $=11.8$ minutes for Y.C13. HRMS (ESI + ): for Tm.C13 found 320.1267; $\left[\mathrm{C}_{44} \mathrm{H}_{57} \mathrm{~N}_{9} \mathrm{O}_{4} \mathrm{Tm}\right]^{3+}$ requires 320.1269; for Y.C13 found 293.4507; [ $\left.\mathrm{C}_{44} \mathrm{H}_{57} \mathrm{~N}_{9} \mathrm{O}_{4} \mathrm{Y}\right]^{3+}$ requires 293.4508.

## Protein expression and purification of ubiquitin S57C

Uniformly ${ }^{15} \mathrm{~N}$-labelled human ubiquitin S 57 C was produced in fusion with a C -terminal SerHis 6 peptide. The protein was expressed from a pETMCSI vector (Neylon et al., 2000) in E. coli BL21 (DE3) in a bioreactor, using a published high cell-density protocol with cells grown in the minimal fermenter medium containing ${ }^{15} \mathrm{NH}_{4} \mathrm{Cl}$ (Klopp et al., 2018). Protein expression was induced with 1 mM isopropyl $\beta$-D-1-thiogalactopyranoside and the cells were harvested by centrifugation after expression at $18^{\circ} \mathrm{C}$ for 16 h . Following resuspension in buffer A ( 50 mM Tris-HCl pH 7.5, $300 \mathrm{mM} \mathrm{NaCl}, 5 \%$ glycerol, 10 mM imidazole), the cells were
lysed using an Avestin Emulsiflex C5 (Avestin, Canada) using two passes at 10,000-15,000 psi. The cell lysate was clarified by centrifugation for 1 h at $30,000 \mathrm{~g}$. The supernatant was loaded onto a 5 mL HisTrap FF column connected to an ÄKTA pure 25 chromatography system (Cytiva, USA). The column was washed with 20 column volumes buffer B (same as buffer A but with 20 mM imidazole) and the protein was eluted with 3 column volumes buffer C (same as buffer A but with 500 mM imidazole). The eluted protein was desalted using a HiPrep Desalting 26/10 column (Cytiva, USA) equilibrated with buffer D ( 50 mM Tris- HCl buffer $\mathrm{pH} 8.0,150 \mathrm{mM}$ sodium chloride, 1 mM dithiothreitol (DTT)) followed by removal of the C-terminal $\mathrm{SerHis}_{6}$ peptide with deubiquitinase (Catanzariti et al., 2004). The final yield of purified ubiquitin S57C was $60 \mathrm{mg} / \mathrm{L}$ cell culture.

## Protein tagging

To a $20 \mu \mathrm{M}$ solution of ${ }^{15} \mathrm{~N}$-labelled ubiquitin S57C in phosphate-buffered saline (PBS) buffer $\left(137 \mathrm{mM} \mathrm{NaCl}, 2.7 \mathrm{mM} \mathrm{KCl}, 10 \mathrm{mM} \mathrm{Na}_{2} \mathrm{HPO}_{4}, 1.8 \mathrm{mM} \mathrm{KH}_{2} \mathrm{PO}_{4}, \mathrm{pH} 7.5\right) 5 \mathrm{mM}$ DTT was added and the sample was incubated at $25^{\circ} \mathrm{C}$ for 1 h . DTT was removed by passing through a PD-10 desalting column (Cytiva, USA) equilibrated with PBS buffer. Immediately following the desalting step, 5 equivalents of the required tag were added and the sample was incubated with shaking at $25^{\circ} \mathrm{C}$ for 16 h . Excess unreacted tag was removed by passing through a PD10 desalting column equilibrated with the NMR buffer ( 20 mM phosphate buffer, pH 6.5 ). Finally, the NMR samples were concentrated using 3 kDa molecular weight cut-off Amicon ultrafiltration centrifugal tubes (Merck Millipore, USA) to a protein concentration of 0.2 mM .

## NMR measurements

All NMR data were acquired at $35^{\circ} \mathrm{C}$ on a Bruker 800 MHz NMR spectrometer equipped with a TCI cryoprobe. The PCSs of the amide protons were measured in $\left[{ }^{15} \mathrm{~N},{ }^{1} \mathrm{H}\right]$-HSQC spectra recorded with acquisition times of $t_{1 \text { max }}=39 \mathrm{~ms}$ and $t_{2 \text { max }}=95 \mathrm{~ms}$. PCSs were measured as the chemical shift measured in the sample with $\mathrm{Tm}^{3+}$ tag minus the corresponding chemical shift in the sample with $\mathrm{Y}^{3+}$ tag.

Table S1. Experimentally measured amide proton PCSs for ubiquitin S57C with C1, C2, C12 and C13 tags. ${ }^{\text {a }}$

| Residue number | $\begin{array}{r} \hline \text { C1 PCS } \\ (\mathrm{ppm}) \end{array}$ | $\begin{array}{r} \text { C2 PCS } \\ (\mathrm{ppm}) \end{array}$ | $\begin{array}{r} \text { C12 PCS } \\ (\mathrm{ppm}) \end{array}$ | C13 PCS (ppm) |
| :---: | :---: | :---: | :---: | :---: |
| 2 | -0.099 | 0.159 | 0.125 | 0.108 |
| 4 | 0.111 | 0.225 |  | 0.456 |
| 5 | 0.119 | 0.172 |  |  |
| 6 | 0.148 | 0.171 | -0.103 | 0.477 |
| 7 | 0.093 | 0.107 | -0.046 | 0.311 |
| 8 | 0.101 | 0.097 | -0.052 | 0.322 |
| 9 | 0.080 | 0.077 | -0.037 | 0.247 |
| 11 | 0.067 | 0.077 | -0.027 |  |
| 12 | 0.055 | 0.078 | -0.012 | 0.203 |
| 13 | 0.094 | 0.128 | 0.001 | 0.315 |
| 14 | 0.062 | 0.112 | 0.070 | 0.242 |
| 15 | 0.088 | 0.186 | 0.128 | 0.346 |
| 17 | 0.072 | 0.278 | 0.550 | 0.333 |
| 18 | 0.272 | 0.437 | 1.456 | 0.457 |
| 20 |  | 0.801 | 4.730 | 0.163 |
| 21 | 0.807 | 0.719 | 2.926 |  |
| 22 |  |  |  | 1.007 |
| 26 | 0.491 |  |  | 1.036 |
| 27 | 0.401 | 0.346 | 0.410 | 0.935 |
| 29 | 0.227 | 0.221 | 0.428 | 0.508 |
| 30 | 0.202 | 0.206 | 0.274 | 0.501 |
| 31 | 0.162 |  | 0.223 | 0.442 |
| 32 | 0.121 | 0.126 | 0.240 | 0.313 |
| 33 | 0.102 | 0.119 | 0.184 | 0.271 |
| 34 | 0.096 | 0.109 | 0.132 | 0.275 |
| 35 | 0.088 | 0.092 | 0.127 | 0.253 |
| 36 | 0.106 | 0.102 | 0.102 | 0.294 |
| 39 | 0.146 | 0.112 |  | 0.411 |
| 40 | 0.142 | 0.113 | 0.063 | 0.420 |
| 41 | 0.180 | 0.142 |  | 0.531 |
| 42 | 0.201 | 0.166 |  | 0.641 |
| 43 | 0.345 | 0.283 |  | 1.145 |
| 44 |  | 0.270 | -0.282 |  |
| 45 | 0.405 | 0.355 | -0.639 | 1.426 |
| 46 | 0.265 | 0.243 | -0.547 | 0.802 |
| 47 | 0.225 | 0.170 | -0.494 | 0.681 |
| 48 |  |  |  | 1.259 |
| 50 | 0.479 | 0.396 | -0.530 | 1.817 |


| 51 | 0.839 | 0.691 | -0.878 | 3.405 |
| ---: | ---: | ---: | ---: | ---: |
| 52 | 0.526 | 0.360 |  | 1.579 |
| 59 |  | 3.224 |  |  |
| 61 | 1.310 | 2.458 |  |  |
| 62 | 0.092 | 0.544 | -0.821 | 0.503 |
| 64 | -0.067 | 0.179 | -0.094 | 0.185 |
| 65 | 0.013 | 0.241 | -0.243 | 0.318 |
| 66 | 0.118 | 0.214 | -0.318 | 0.462 |
| 67 | 0.198 | 0.254 | -0.178 | 0.663 |
| 68 | 0.251 | 0.254 | -0.273 | 0.826 |
| 69 | 0.165 | 0.166 | -0.112 |  |
| 70 | 0.187 | 0.164 | -0.118 | 0.600 |

${ }^{\text {a }}$ The PCSs were measured in $\left[{ }^{1} \mathrm{H},{ }^{15} \mathrm{~N}\right]$-HSQC spectra of uniformly ${ }^{15} \mathrm{~N}$-labelled ubiquitin S 57 C labelled with one of the four different tags (loaded with $\mathrm{Tm}^{3+}$ or $\mathrm{Y}^{3+}$ in the paramagnetic and diamagnetic tags, respectively).

Table S2. Fitted $\Delta \chi$-tensor parameters for ubiquitin S57C with C1, C2, C12 and C13 tags. ${ }^{\text {a }}$

| Tag | $\Delta \chi_{\mathrm{ax}^{\mathrm{a}}}$ | $\Delta \chi_{\mathrm{rh}^{\mathrm{a}}}{ }^{\mathrm{a}}$ | $x(\AA)$ | $y(\AA)$ | $z(\AA)$ | $\alpha\left({ }^{\circ}\right)$ | $\beta\left({ }^{\circ}\right)$ | $\gamma\left({ }^{\circ}\right)$ | $Q^{\mathrm{b}}$ | $d(\AA)^{\mathrm{c}}$ |
| :--- | ---: | :---: | :---: | :--- | ---: | :---: | :---: | ---: | :---: | :---: |
| C1 | 8.2 | 2.2 | 18.729 | 11.036 | 11.086 | 48 | 61 | 124 | 0.019 | 7.6 |
| C2 | 7.0 | 3.0 | 15.662 | 13.465 | 11.630 | 38 | 73 | 1 | 0.020 | 8.3 |
| C12 | 20.1 | 10.3 | 20.326 | 10.342 | 9.039 | 9 | 93 | 66 | 0.024 | 7.8 |
| C13 | 18.0 | 4.0 | 17.797 | 13.067 | 15.921 | 47 | 76 | 13 | 0.017 | 8.3 |

${ }^{\text {a }} \Delta \chi$ tensor values are given in units of $10^{-32} \mathrm{~m}^{3}$. Coordinates and Euler angles are reported with respect to the PDB coordinates 1UBQ (Vijay-Kumar et al., 1987). Euler angles are given in the "ZYZ" convention. Tags were loaded with $\mathrm{Tm}^{3+}$ or $\mathrm{Y}^{3+}$ ions to obtain paramagnetic samples and diamagnetic references, respectively.
${ }^{\mathrm{b}} Q$ factor calculated as root-mean-square deviation between measured and predicted PCSs divided by the root-mean-square of the measured PCSs.
${ }^{c}$ Distance of the paramagnetic centre from the $C^{\beta}$ atom of residue 57 in the crystal structure 1UBQ.


Figure S1. Correlation between experimental and back-calculated PCSs of backbone amide protons in ubiquitin S57C for the $\Delta \chi$-tensor fits of Table S1.


Figure S2. PCS parallel metric versus volume of the localisation space for different numbers of tagging sites and tags. The tagging sites and tag distributions were modelled as described in the main text, with the SoI located at a distance $d$ of $20 \AA$ from the paramagnetic centres (Fig. 1). Each plot shows the result of 1000 sampled $\Delta \chi$-tensor orientations. The correlation between the metric and the localisation spaces improves with the number of tags and tagging sites.


Figure S3. PCS perpendicular metric versus volume of the localisation space. The calculation used the same tag geometries and tag distributions as in Fig. S3.

Table S3. RMSD threshold and characterisation of the localisation spaces of the amide protons shown in Fig. 7 of the main text.

| Segment | Residue <br> number | RMSD <br> threshold <br> $(\mathrm{ppm})^{\mathrm{a}}$ | $V_{\text {RMSD }}$ <br> $\left(\AA^{3}\right)^{\mathrm{b}}$ | Electron- <br> nuclear <br> distance $(\AA)^{\mathrm{c}}$ | Distance of <br> nucleus from <br> PCS RMSD <br> minimum $(\AA)^{\mathrm{d}}$ | Dot- <br> product <br> metric | Cross- <br> product <br> metric |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | 12 | 0.003 | 0.19 | 32 | 0.36 | 0.63 | 0.008 |
| A | 13 | 0.004 | 0.18 | 28 | 0.54 | 0.64 | 0.014 |
| A | 14 | 0.003 | 0.18 | 28 | 0.52 | 0.68 | 0.013 |
| A | 15 | 0.007 | 0.14 | 23 | 0.35 | 0.66 | 0.026 |
| A | 17 | 0.02 | 0.47 | 19 | 0.78 | 0.65 | 0.056 |
| B | 29 | 0.01 | 0.19 | 23 | 0.32 | 0.70 | 0.035 |
| B | 30 | 0.008 | 0.30 | 25 | 0.54 | 0.69 | 0.028 |
| B | 32 | 0.008 | 0.22 | 28 | 0.75 | 0.73 | 0.016 |
| B | 33 | 0.006 | 0.23 | 29 | 1.2 | 0.73 | 0.014 |
| B | 34 | 0.004 | 0.26 | 30 | 1.1 | 0.72 | 0.011 |
| B | 35 | 0.006 | 0.26 | 32 | 0.62 | 0.72 | 0.010 |
| B | 36 | 0.005 | 0.25 | 31 | 1.0 | 0.68 | 0.010 |
| C | 64 | 0.023 | 0.49 | 19 | 0.72 | 0.60 | 0.046 |
| C | 65 | 0.02 | 0.29 | 18 | 0.28 | 0.64 | 0.054 |
| C | 66 | 0.013 | 0.18 | 20 | 0.89 | 0.74 | 0.040 |
| C | 67 | 0.013 | 0.14 | 21 | 0.55 | 0.73 | 0.034 |
| C | 68 | 0.008 | 0.10 | 22 | 0.43 | 0.84 | 0.031 |

${ }^{\text {a }}$ Lowest contour level of the localisation spaces (referred to as $\delta_{\text {RMSD }}^{\text {thresh }}$ in the main text) used to plot the regions shown in Fig. 7 of the main text.
${ }^{\text {b }}$ Volume of the localisation space associated with the PCS RMSD threshold given in the preceding column.
${ }^{c}$ Distance of the amide proton from the average $\Delta \chi$-tensor position.
${ }^{d}$ Distance of the amide proton in the crystal structure from the position with the minimal PCS RMSD.

Table S4. Angles at which pairs of PCS isosurfaces intersect at the amide protons shown in Fig. 7 of the main text. ${ }^{\text {a }}$

| Segment | Residue | C1 vs C2 | C1 vs C12 | C1 vs C13 | C2 vs C12 | C2 vs C13 | C12 vs C13 |
| :---: | :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| A | Thr12 | 27 | 96 | 14 | 84 | 15 | 96 |
| A | Ile13 | 25 | 96 | 11 | 87 | 17 | 99 |
| A | Thr14 | 34 | 81 | 17 | 69 | 21 | 83 |
| A | Leu15 | 36 | 82 | 15 | 74 | 26 | 88 |
| A | Val17 | 47 | 76 | 19 | 68 | 38 | 88 |
| B | Lys29 | 20 | 73 | 19 | 74 | 22 | 91 |
| B | Ile30 | 20 | 78 | 15 | 76 | 20 | 91 |
| B | Asp32 | 19 | 70 | 15 | 67 | 19 | 83 |
| B | Lys33 | 22 | 71 | 14 | 66 | 19 | 81 |
| B | Glu34 | 20 | 74 | 12 | 68 | 17 | 82 |
| B | Gly35 | 18 | 73 | 12 | 68 | 16 | 81 |
| B | Ile36 | 16 | 80 | 11 | 75 | 15 | 87 |
| C | Glu64 | 54 | 81 | 20 | 84 | 34 | 84 |
| C | Ser65 | 44 | 99 | 15 | 104 | 29 | 101 |
| C | Thr66 | 29 | 120 | 11 | 116 | 18 | 118 |
| C | Leu67 | 23 | 117 | 5 | 113 | 20 | 120 |
| C | His68 | 15 | 131 | 2 | 125 | 15 | 132 |

${ }^{a}$ In degrees. The angles were calculated between the PCS gradient vectors. They differ from the angle score of Zimmermann et al. (2019) by $90^{\circ}$.

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